COMPARATIVE STUDY OF THE ANALYSIS TECHNIQUES OF MAT FOUNDATION

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নং 86998
তাং 9.12.93
লাইব্রেরী, ঢাকা

BY

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A thesis submitted to the Department of Civil Engineering of
Bangladesh University of Engineering and Technology, Dhaka in
partial fulfilment of the requirements for the degree

of

MASTER OF SCIENCE IN CIVIL ENGINEERING

#86998#

November, 1993

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DECLARATION

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ACKNOWLEDGEMENT

The author wishes to express his sincerest gratitude to Dr. Salek M. Seraj, Assistant Professor of Civil Engineering Department, BUET for his continuous guidance, invaluable suggestions and affectionate encouragement at every stage of this study.

Among others, acknowledgements are made to Mr. M.A. Malek for typing the thesis and Mr. M. Shahiduddin for his assistance in drafting some of the figures presented in this thesis.

SYNOPSIS

Mat foundations are becoming increasingly popular in the construction of high-rise buildings as they are usually found to be economical in cases where the base soil has a low bearing capacity and/or the column loads are very large. An alternative to mat is pile foundation which is not only expensive but also causes problem to rail tunnel construction.

There are several methods available for analysis of mat foundation; all such techniques i.e. conventional method, ACI approximate flexible method, and discrete element methods developed so far are based on several assumptions and idealisations of mat on soil. In this study, a detailed comparative study of the performance of these analysis techniques has been carried out. Also the effects of several parameters such as modulus of subgrade reaction of soil, mat thickness, column number and spacing, column loads and their combination, Poisson's ratio and strength of concrete on the behaviour of mat foundation have been studied.

Conventional method of analysis has been found to be inadequate to give realistic results. It has been revealed that ACI approximate flexible method yields moments and deflections comparable to discrete element methods at points away from edges, but in the vicinity of edges, the method has been found to be inadequate. The two discrete element methods i.e. finite difference and finite grid method yield moments and deflections which are comparable to each other.

It has been found that maximum moments at column points are due to finite grid method with soil spring zoning. Finite grid method without zoning, finite difference method and ACI approximate flexible method yield next higher values. Column point moments obtained by conventional method are very small compared to other methods. As for moments between column points, the maximum among comparable methods is due to finite grid method without soil spring zoning. The finite grid method with zoning, finite difference method and ACI approximate flexible method yield next higher moments respectively.

Finite grid method without zoning and finite difference method have been found to give similar deflections at all the points. With soil spring zoning, the deflections towards the centre have been found to be much higher. Soil spring zoning has been proven to simulate mat response to load more realistically.

Mat deflections have been found to decrease almost exponentially with an increase in modulus of subgrade reaction. A slight increase in moment at column points has been noticed with an increase in modulus of subgrade reaction. With an increase in mat thickness, both moments at column points and deflections have been found to be reduced. Moments in both directions at column points have shown an increase with an increase in Poisson's ratio but it has demonstrated no effect on deflections. Both moments at column points and deflections are reduced with an increase of concrete strength. It appeared that keeping stiffness of the mat-soil system unchanged, change in external loads results in a corresponding change in deflections and moments accordingly, as it should in a linear analysis. It has been revealed that if spans remain unchanged in a direction of mat, the summation of moments at column points in that direction remains unchanged no matter how many parallel column lines are present.

Apart from identifying the merits and demerits of different analysis techniques of mat, efforts have been made to put forward rational explanations of the findings of the present study.

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NOTATION

A	Plan area of mat
$A_{\lambda x}$	Coefficient used in equation (2.13a)
B	Breadth of finite grid element
B, b	Width of footing
$B_{\lambda x}$	Coefficient used in equation (2.13b)
B_c	Spacing of columns
B_s	Width of strip in conventional method
$C_{\lambda x}$	Coefficient used in equation (2.13c)
C_E	Euler's constant
D	Flexural rigidity of mat
$D_{\lambda x}$	Coefficient used in equation (2.13d)
DL	Dead load
DQ_e	Average stress increase for a point on the edge of a square plate
DQ_i	Average stress increase at a point where k'_{si} is to be calculated
E	Modulus of elasticity of material used in structure
$E_b I_b$	Flexural rigidity of footing in equation (2.35)
E_c	Modulus of elasticity of concrete
E_s	Modulus of elasticity of soil
e_x, e_y	Eccentricity of resultant vertical loads with respect to centroidal axes of mat
F	Load factor
f'_c	28 day cylinder strength of concrete
G	Shear modulus
H	Vertical projection of finite grid element
H_b	Height of point where pressure is to be calculated
h	Vertical grid size in finite difference method
I_B	Moment of inertia of structure per unit length
I_F	Moment of inertia of footing
I_x, I_y	Moment of inertia of plan area of mat
J	Polar moment of inertia
K_r	Relative stiffness factor
k'_{si}	Modified value of subgrade reaction using zoning
k_s	Modulus of subgrade reaction

k_{v1}	Modulus of subgrade reaction based on 1 ft square bearing plate
L	Length of footing in equation (3.1)
L	Length of finite grid elements
L	Radius of effective stiffness in ACI method
LL	Live load
L_s	Length of strip in conventional method
M_e	Moment correction due to end conditioning forces
M_r, M_t	Radial and tangential moment in ACI approximate method
M_x, M_y	Moment around y and x axes, respectively, as in Fig.2.1
P, p	Concentrated loads
p_s	Wind pressure on surface of building
Q	Shear force
Q_{av}	Average load on strip in conventional method
Q_e	Effect on shear force due to end conditioning force
q	Soil pressure
q_a	Allowable bearing capacity
q_{CBR}	Pressure at 0.25 cm penetration in CBR test
q_u	Unconfined compressive strength
q_{ult}	Ultimate bearing capacity
q_v	Pressure at a depth Z in equation (3.1)
$\overline{q}_{av}$	Modified average soil pressure
q_0	Pressure at surface in equation (3.1)
R	Resultant of vertical column loads
r	Radial distance of point under consideration from point load in ACI method
rh	Horizontal grid size in finite difference method
SF	Safety factor
t	Thickness of mat
U	Total factored load
V	Vertical projection of finite grid element
v	Velocity of maximum probable wind
WL	Wind load
w	Deflection of mat
w_e	Effect on deflection due to end conditioning forces
X_{max}	Maximum soil deflection
x, y	Co-ordinates of point where soil pressure is desired in conventional method

Z1, Z2, Z3, Z4	Functions for plate on soil springs as used in ACI method
Ω	Adjust factor in finite grid method
α	Angle of inclination of finite grid element with horizontal
ϕ	Angle of inclination of point with x axis in ACI method
λ	Characteristic length
δ	Deflection in plate bearing test
μ	Poisson's ratio
ε_{max}	Peak strain in triaxial compression test
[A]	Bridging matrix between [P] and [F] matrices
[e]	Internal member deformation matrix
[F]	Internal member force matrix
[P]	External nodal force matrix
[S]	Bridging matrix between [F] and [e] matrices
[X]	External nodal displacement matrix

CHAPTER 1

INTRODUCTION

1.1 GENERAL

All engineered construction resting on earth must be carried by a foundation which transmits to, and into, the underlying soil or rock the loads coming from the superstructure and its self-weight. There are various types of foundations of a building, the choice of which depends on the building itself and the underlying soil. A mat or raft foundation may be used where the base soil has a low bearing capacity and/or the column loads are so large that more than 50 percent of the area is covered by conventional spread footing.

A mat foundation is a large concrete slab used to interface one or more columns in several lines with base soil. Actually there is no guide line as to when the dimensions of a spread footing makes the transition into being called a mat. Fig.1.1 illustrates several mat configurations as might be used for buildings. The most common type of mat used in construction is the flat plate type shown in Fig.1.1(a). Mats may also be supported by piles in situations such as high ground water to control buoyancy or where the base soil is susceptible to large settlements.

An alternative to mat is pile foundation which can be employed under similar conditions of heavy column loads and low bearing capacity of soil. Pile foundations are not only expensive but also cause problem to underground rail tunnel construction which is likely in a built up urban area.

Mat foundations are also popular for deep basements both to spread the column loads to a more uniform pressure distribution and to provide the floor slab for the basement. A particular advantage for basements at or below the ground water table is to provide a water barrier. Mat foundations are also advantageous in places where settlements may be a problem as where a site contains erratic deposits or lenses of compressible materials etc. The settlement tends to be controlled via:

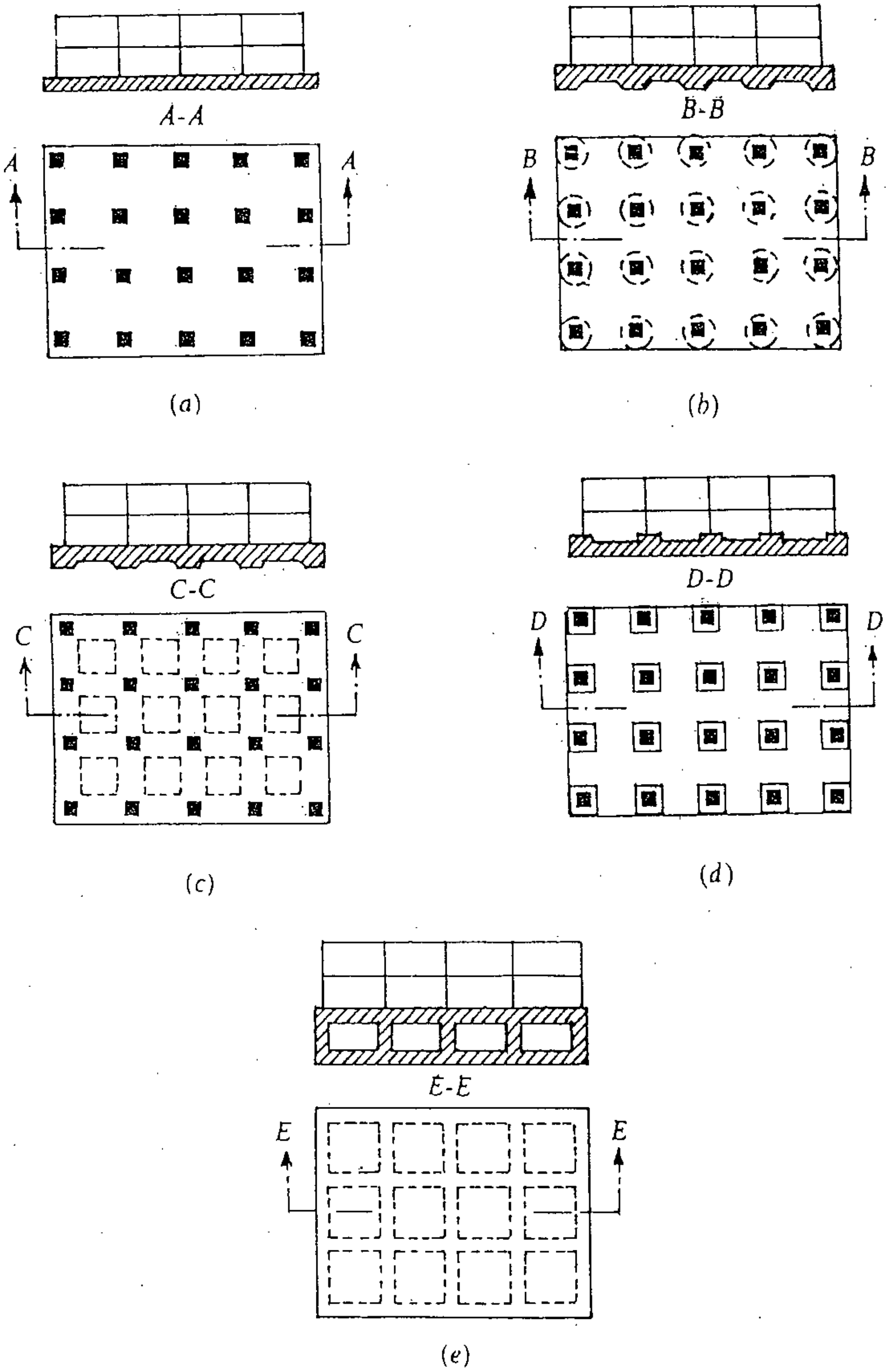

Fig. 1.1 Common types of mat foundations. (a) Flat plate; (b) plate thickened under columns; (c) waffle-slab; (d) plate with pedestals; (e) basement wall as part of mat.

(1) Lower soil contact pressure
(2) Floatation effect : Theoretically if the weight of excavation equals the combined weight of the structure and mat, the system floats in the soil mass and no settlement occurs.
(3) Bridging effect due to
    a. Mat rigidity
    b. Superstructure rigidity contribution to the mat.
(4) Allowing somewhat larger settlements.

Again mat continuity results in a somewhat less amount of differential settlement of the whole structure. For all these advantages, mat foundations are becoming popular every day.

It has been found that mat foundations tend to be heavily over designed. The main reason for over design is the additional cost involved in the analysis because rigorous analysis requires computer and programming tools. Also the designer generally goes for extra margin of safety provided by the over design because usually the cost of over design of mat is reasonable compare to the total cost of the project.

1.2 BACKGROUND OF THE STUDY

Several methods are available to analyse and design mat foundations. The most simplified method is the conventional method which considers the mat strip by strip loaded by a line of columns and resisted by soil pressure. Each strip is then analysed as a combined footing having linearly varying soil pressure acting under it. American Concrete Institute (ACI) Committee 336 (1966) proposed that this method can be used for design if the adjacent spans and column loads do not vary by more than 20 percent. Conventional method [Teng (1965), Bowles (1974), Bowles (1966), Arora (1992)] assumes the mat to be rigid which is not true for most mats. Moreover, when the mat is divided into strips, it loses its plate nature i.e. bending in both the directions. ACI Committee 336 (1966) also proposes a more general approach in which the mat is assumed flexible and the plate action is also considered. This method, known as ACI approximate flexible method, utilises functions, first introduced by Schleicher (1926) for circular plates on Winkler's medium, which models soil as springs. The effect of a concentrated load on plate damps out quite rapidly and as a result there is a zone of influence for each column which is circular in shape. To find the effect at a point for all the loads in a mat, the first step is to find the column loads whose influence zone extends beyond that point. By superimposing the effect of all these column loads, the total effect at the point can be determined. But the problem arises when the zone of influence of a column extends beyond the edge of the mat and then the mat is divided into strips in both directions and corrections are applied assuming strips to be semi-infinite beams. Shukla (1984) presented design aides using this method and Mician (1985) in the discussion of Shukla's method pointed some corrections and presented another example of a mat problem.

A more refined approach to mat solution is to use discrete element methods where the mat is divided into elements by gridding. Finite difference method is one technique to solve such a problem where the differential equation of the plate on soil is solved through the use of difference equations. Bowles (1988) proposed finite grid method to analyse mat which uses techniques of finite element. In this method the stiffness matrix of the whole mat-soil system is formed using beam-column elements along with soil springs. Nodal displacements and member forces can be found as the stiffness matrix of the whole mat-soil system and load matrix are known. ACI approximate flexible method and finite grid method are based on Winkler's medium and soil springs are calculated using modulus of subgrade reaction of soil. There is also finite element method of the elastic continuum where modulus of elasticity and Poisson's ratio are used as soil parameters. Bowles asserts that there is actually little computational evidence that the finite element method of elastic continuum provides better solutions than using Winkler foundation.

1.3 OBJECTIVE OF THE PRESENT RESEARCH

The general objectives of the research are:

(a) To develop computer programs using methods of analysis of mat foundations which are not available and to adopt available programs after modifications, if needed, and to incorporate automatic data generation feature, if necessary.

(b) To use these programs to analyse a certain selected mat and to compare the results obtained by them.

(c) To study the effects of several parameters such as modulus of subgrade reaction of soil, mat thickness, column number and spacing, column loads and their combination, Poisson's ratio and strength of concrete.

(d) To assess the comparative merits and demerits of different analysis methods.

1.4 METHODOLOGY

In an attempt to compare the analysis techniques, a survey of the related literature has been made. All the available analytical approaches are studied in details in order to delineate the difference between them. Also the material properties required in the analysis are discussed in details.

Computer program using different analysis techniques are available in some texts and they are adopted to use on microcomputers after modification and after incorporating data generation schemes. Computer programs are developed for those methods which are not yet computerised or whose programs are not available. As soon as the programs are ready, they can be used as analytical tools to gain insight into the behaviour of different mat-soil systems.

Comprehensive analysis of a prototype mat foundation has been made using all the computer programs employing different methods. Also the sensitivity analyses of geometric parameters and material properties on the overall behaviour of the mat have been carried out.

A review of the available analytical methods are presented in chapter 2. The main features of computer programs i.e. input, output, data generation etc. are discussed in chapter 3. A planned scheme defining the mat problem to be used in the comparative study of all the methods and different parameters involved in mat analysis is presented in chapter 4. In chapter 5, a comparative study of results obtained by different methods and the results of a parametric study are discussed. Conclusions and recommendation for future research are presented in chapter 6.

CHAPTER 2

ANALYSIS METHODS

2.1 INTRODUCTION

There are several methods available to analyse and design a mat foundation:

a) Conventional method
b) ACI approximate flexible method
c) Discrete element methods, such as
    Finite difference method,
    Finite grid method and
    Finite element method

In the conventional method, the mat is divided into strips loaded by a line of columns and resisted by soil pressure. Each strip is then analysed as a combined footing having linearly varying soil pressure acting under it. ACI committee 336 (1966) continues to suggest an approximate flexible method in which the mat is assumed to be flexible rather than rigid in contrast to conventional method. This approximate flexible method is based on the theories of circular plate on Winkler medium and considers the mat as a whole using method of superposition.

All the discrete element methods discussed in this chapter are based on Winkler medium. Here the mat is divided into discrete elements by gridding and the elements are supported on soil springs. The finite difference method is based on the fourth order differential equation for plates and the method involves solving this differential equation through finite difference equations. Finite grid method uses the techniques of finite element method. In the finite grid method, stiffness of the whole mat and soil springs is formed from beam-column element stiffnesses. Nodal displacements and member forces can be found as the global stiffness of the mat-soil system and load matrix are known.

The most important soil parameter required for Winkler medium, the modulus of subgrade reaction, is also discussed in this chapter.

2.2 CONVENTIONAL METHOD OF ANALYSIS

In the conventional method of design, the mat is assumed to be infinitely rigid and the pressure distribution is taken planar i.e. linearly varying. Here the mat is divided in both directions into strips loaded by column loads and supported by soil pressure. This method can be used where the mat is very rigid and column spacing is fairly uniform in both spacing and loads.

2.2.1 Analysis Procedure

The procedure for the conventional analysis consists of the following steps:

(1) The line of action of all loads acting on the mat is determined.
(2) The contact pressure distribution is calculated using the equation:

$$q = R\left(\frac{1}{A} \pm \frac{e_x x}{I_y} \pm \frac{e_y y}{I_x}\right) \tag{2.1}$$

where,

R $= \sum p_i$ = resultant of vertical column loads
A = plan area of mat
I_x, I_y = moment of inertia of plan area of mat with respect to x and y axes.
e_x, e_y = eccentricity of resultant vertical loads with respect to the centroidal axes
x, y = co-ordinate locations where soil pressures are desired.

The maximum soil pressure should be less than the allowable bearing capacity.

(3) The slab is divided into strips in x and y directions. Each strip is assumed to act as an independent beam subjected to the contact pressure and the column load. Contact pressure is taken as the average of two end pressures of the strip.

(4) This average contact pressure of the strip and the column loads should be modified since they do not satisfy statics. The modification is done as follows:

a) The average load on the strip is calculated by

$$Q_{av} = \frac{1}{2}\text{ (downward load + upward force)} \tag{2.2}$$

where,

downward load = sum of all column loads
upward loads = soil reaction based on average contact pressure calculated in step (3)

b) The modified average soil pressure is calculated by

$$\bar{q}_{av} = \frac{Q_{av}}{L_s * B_s} \tag{2.3}$$

where,

L_s , B_s= Length and breadth of strip

(c) The modified column load is determined by multiplying each load in strip by a factor,

$$F = \frac{Q_{av}}{\sum Column\ loads} \tag{2.4}$$

(5) The shear force and bending moment diagrams for the strip are drawn using modified column load and modified average soil pressure $\bar{q}_{av}$.

(6) The individual strips are designed for the bending moment and shear force found in step 5.

Arora (1992) proposes that as the analysis is approximate, the actual reinforcement provided should be twice the computed value.

2.2.2 Applicability and Limitations

According to the ACI Committee 336 (1966), the design of mats may be accomplished using the conventional method if

(1) The average of the two adjacent spans in a continuous strip is less than 1.75/λ (λ is defined in equation 2.12) and adjacent column load and column spacing do not vary by more than 20 percent of the greater value.

(2) The relative stiffness factor K_r is found to be greater than 0.5, where

$$K_r = \frac{E.I_B}{E_S.b^3} \tag{2.5}$$

where,

E = modulus of elasticity of materials used in structure, kips per sq. ft.

I_B = moment of inertia of structure per unit length, ft^3

I_F = Moment of inertia of footing, ft^3

E_S = Modulus of elasticity of soil kips per sq. ft.

b = Width of footing, ft

An approximate value of EI_B per unit length of building can be found by summing flexural rigidity of footing (EI_F), the flexural rigidity of each framed member (EI_b) and the flexural rigidity of any shear walls ($Eah^3/12$) where a and h are the thickness and height of the walls, respectively; EI_B is given by:

$$EI_B = EI_F + \sum EI_b + \sum \frac{Eah^3}{12.} \tag{2.6}$$

2.3 ACI APPROXIMATE FLEXIBLE METHOD

ACI Committee 336 (1966) suggested this method for the general case of a flexible mat supporting columns at random locations with varying intensities of load. This procedure is based on theories of circular plate on Winkler medium. Shukla (1984) recommends this method to calculate moment, shear forces and deflections of a mat with the help of charts provided by him.

The effect of a concentrated load on a typical mat has been found to be damped out quite rapidly. It is, therefore, possible to consider the mat as a plate and determine the effect of a column load in the area surrounding the load. By superimposing the effect of all the column loads within the zone of influence, the total effect of all the column loads at any point is determined. This zone of influence is generally not large and it is suggested that it is not necessary to consider columns further than two bays in all directions to determine stresses at a particular point in most cases. Since the effect of each load is transmitted through the mat in a radial direction, the use of polar co-ordinates is necessary.

2.3.1 Analysis Procedure

The following procedure is recommended .

(1) Overall thickness of mat is calculated from shear requirements at critical section, (ACI 318-83, section 11.11).

(2) The modulus of subgrade reaction is established (discussed later).

(3) The flexural rigidity of the mat D in lb-inch is calculated as follows

$$D = \frac{E_c t^3}{12(1-\mu^2)} \tag{2.7}$$

where,

E_c = Modulus of elasticity of concrete
= $57000\sqrt{f'_c}$, psi (as per ACI 318-83, Section 8.5.1)
t = thickness of mat, inch
μ = Poisson's ratio of concrete (0.15 to 0.25)

(4) The radius of effective stiffness L is then calculated as follows:

$$L = \sqrt[4]{\frac{D}{k_s}} \tag{2.8}$$

where,

k_s = modulus of subgrade reaction

(5) Radius of influence is 4L.

(6) The radial and tangential moments, the shear and deflection at a point are calculated using the following formulas:

$$M_r = -\frac{P}{4}\left[Z_4\left(\frac{r}{L}\right) - (1-\mu)\frac{Z_3'\left(\frac{r}{L}\right)}{\frac{r}{L}}\right] \tag{2.9a}$$

$$M_t = -\frac{P}{4}\left[\mu Z_4\left(\frac{r}{L}\right) + (1-\mu)\frac{Z_3'\left(\frac{r}{L}\right)}{\frac{r}{L}}\right] \tag{2.9b}$$

$$Q = -\frac{P}{4L} Z_4'\left(\frac{r}{L}\right) \tag{2.9c}$$

$$w = \frac{pL^2}{4D} Z_3\left(\frac{r}{L}\right) \tag{2.9d}$$

where,

r = distance of point under investigation from point load along radius.

L = radius of effective stiffness

M_r, M_t = radial and tangential moments (polar co-ordinates) for a unit width of mat

p = concentrated load

Z_3, Z'_4, Z_4 = functions first introduced by Schleicher(1926) which can be found in Table III of Hetenyi (1946).

(7) To convert radial and tangential moments to rectangular co-ordinates, (Fig.2.1) the following formula are used:

$$M_x = M_r \cos^2 \phi + M_t \sin^2 \phi \tag{2.10a}$$
$$M_y = M_r \sin^2 \phi + M_t \cos^2 \phi \tag{2.10b}$$

(8) For the combination of responses due to all the column loads, the effects of individual columns should be superimposed at points of interest.

(9) If the edge of the mat is at a distance less than 4L from an individual column load, a correction should be applied as follows:

a) Moment, shear, and deflection at the edge of the mat due to column loads within the radius of influence should be calculated as in step 6-8.

b) The mat may be divided into strips of unit width in both directions.

c) Assuming the strips as semi-infinite beams, shear, and moment equal and opposite to those obtained in step (a) should be applied and their effects at various points superimposed on the respective values obtained in step 8.

d) For moment, shear and deflection in a semi infinite beam, the following relationships are used:

$$M_e = M_1 A_{\lambda x} - \frac{P_1}{\lambda} B_{\lambda x} \tag{2.11a}$$

$$Q_e = -2 M_1 \lambda B_{\lambda x} - P_1 C_{\lambda x} \tag{2.11b}$$

$$w_e = -\frac{2 M_1 \lambda^2}{k_s} C_{\lambda x} + \frac{2 P_1 \lambda}{k_s} D_{\lambda x} \tag{2.11c}$$

where,

M_1,P_1= moment and shear, respectively from step a

$$\lambda = \sqrt[4]{\frac{k_s b}{4 E_c I_b}} \tag{2.12}$$

b = width of mat strip, 1 ft.

I_b = moment of inertia of mat strip.

$$A_{\lambda x} = e^{-\lambda x}(cos\lambda x + \sin \lambda x) \tag{2.13a}$$

$$B_{\lambda x} = e^{-\lambda x} \sin \lambda x \tag{2.13b}$$

$$C_{\lambda x} = e^{-\lambda x}(cos\lambda x - \sin \lambda x) \tag{2.13c}$$

$$D_{\lambda x} = e^{-\lambda x} \cos \lambda x \tag{2.13d}$$

2.3.2 The Z Functions

The Z functions have the characteristic features of exponential waves; Z_1 and Z_2 increase rapidly with the increase of their argument while Z_3 and Z_4 decrease as the argument ($x = \frac{r}{L}$) increases as shown in Fig.2.2. They can be written in the form of power series as follows:

$$Z_1(x) = 1 - \frac{(x/2)^4}{(2!)^2} + \frac{(x/2)^8}{(4!)^2} - \frac{(x/2)^{12}}{(6!)^2} + ... \tag{2.14a}$$

$$Z_2(x) = -\frac{(x/2)^2}{(1!)^2} + \frac{(x/2)^6}{(3!)^2} - \frac{(x/2)^{12}}{(5!)^2} + ... \tag{2.14b}$$

$$Z_3(x) = \frac{1}{2} Z_1(x) - \frac{2}{\pi}[R_1(x) + Z_2(x) \log\{\gamma^* \frac{x}{2}\}] \tag{2.14c}$$

$$Z_4(x) = \frac{1}{2} Z_2(x) + \frac{2}{\pi}[R_2(x) + Z_1(x) \log\{\gamma^* \frac{x}{2}\}] \tag{2.14d}$$

where

$\log \gamma^* = C_E$ = Euler's constant = 0.577216

$$R_1(x) = (x/2)^2 - \frac{\phi(3)}{(3!)^2}(x/2)^6 + \frac{\phi(5)}{(5!)^2}(x/2)^{10} - ... \tag{2.15a}$$

$$R_2(x) = \frac{\phi(2)}{(2!)^2}(x/2)^4 - \frac{\phi(4)}{(4!)^2}(x/2)^8 + \frac{\phi(6)}{(6!)^2}(x/2)^{12} - .. \tag{2.15b}$$

$$\phi(n) = \sum_{t=1}^{n} \frac{1}{t} \tag{2.15c}$$

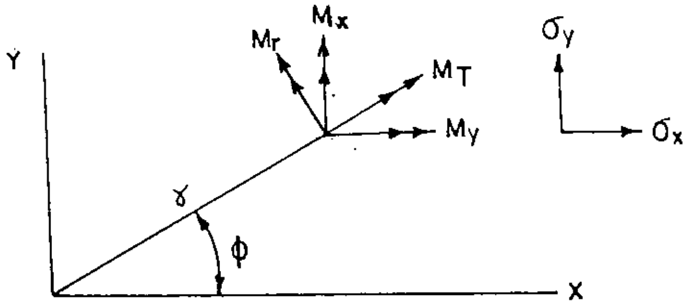

Fig. 2.1 Conversion of radial and tangential moments to cartesian co-ordinates.

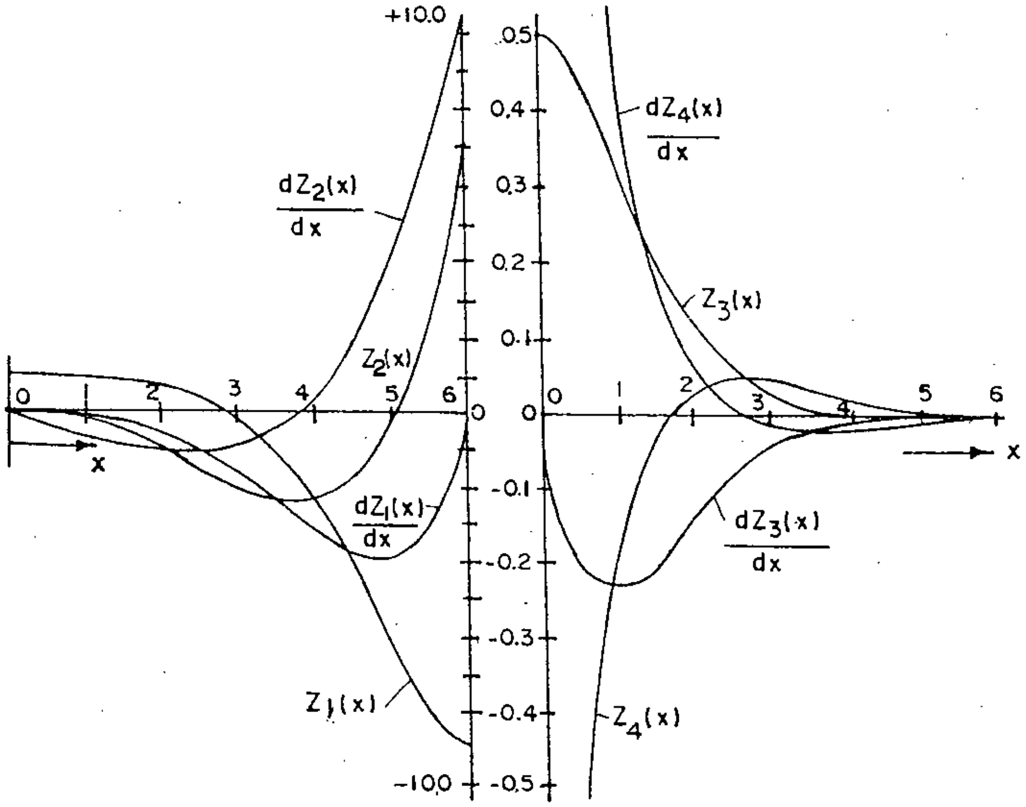

Fig. 2.2 Variation of Z functions with x [After Hetenyi (1946)]

2.4 FINITE DIFFERENCE METHOD

The finite difference solution holds considerable promise for mat analysis. This approach has been used [Deryck and Severn (1960,1961), Severn (1966), Bowles (1968)] to analyse large flat slabs on elastic medium. The finite difference solution utilises thin plate theory, but when the plan dimensions are reasonable compared to the thickness, the error in neglecting the plate thickness is very small.

The finite difference method uses the fourth order differential equation

$$\frac{\delta^4 w}{\delta x^4}+\frac{2\delta^4 w}{\delta x^2 \delta y^2}+\frac{\delta^4 w}{\delta y^4}=\frac{q}{D}+\frac{P}{D(\delta x\, \delta y)} \tag{2.16}$$

where,

w = deflection

q = subgrade reaction per unit area of mat

$D=\dfrac{E_c t^3}{12(1-\mu^2)}$ = flexural rigidity of mat

E_c = modulus of elasticity

t = thickness of mat

μ = Poisson's ratio

which can be transposed into a finite difference equation when r=1 (Fig.2.3)

$$\begin{aligned}&20w_O-8(w_T+w_B+w_R+w_L)+2(w_{TL}+w_{TR}+w_{BL}+w_{BR})\\&+(w_{TT}+w_{BB}+w_{LL}+w_{RR})=\frac{qh^4}{D}+\frac{Ph^2}{D}\end{aligned} \tag{2.17}$$

and when $r \neq 1$, this becomes,

$$\begin{aligned}&\left(\frac{6}{r^4}+\frac{8}{r^2}+6\right)w_O+\left(-\frac{4}{r^4}-\frac{4}{r^2}\right)(w_L+w_R)+\left(-\frac{4}{r^2}-4\right)(w_T+w_B)\\&+\frac{2}{r^2}(w_{TL}+w_{TR}+w_{BL}+w_{BR})+w_{TT}+w_{BB}+\frac{1}{r^4}(w_{LL}+w_{RR})=\frac{qrh^4}{rD}+\frac{Ph^2}{rD}\end{aligned} \tag{2.18}$$

The soil pressure is based on the concept of subgrade reaction k_s resulting in

$$q = -k_s w_o \tag{2.19}$$

Carrying this deflection term to the left of equation (2.17), which is for $r = 1$, results in an increase of the term $20w_o$ to

$$20w_O + \frac{k_s h^4}{D} w_O$$

and when $r \neq 1$, w_o term of equation (2.18) can be rearranged to

$$\left(\frac{6}{r^4} + \frac{8}{r^4} + 6 + \frac{k_s h^4}{D}\right) w_O$$

For a given mat, one difference equation can be written for each point of intersection. By solving these simultaneous equations, the deflections at all points can be determined. Consequently soil reaction, bending moment and shears can be found.

2.5 FINITE GRID METHOD

Finite grid method uses the finite element technique. Here the entire mat is divided into beam-column elements by suitable gridding. Stiffness matrix for each element with two soil springs at the ends is built and at the same time a global matrix for the whole mat on soil is developed. When the global stiffness and the load matrix are known, the node deflection matrix can be readily obtained and hence the member force and nodal displacement can be found.

The method is described below:

2.5.1 General Equation in Solution

At any node (junction of two or more members) on the structure (Fig.2.4) ,

$$P_i = A_i F_i \tag{2.20}$$

which states that the external node force P is equal to the internal member forces F using bridging constants A. It is understood that P and F are used for either moment or forces and that this equation is shorthand notation for several values of $A_i F_i$ summed to equal the ith nodal force.

For the full set of nodes of any structure and using matrix notation where [P], [F] are column vectors and [A] is a rectangular matrix, this becomes

$$[P] = [A][F] \tag{2.21a}$$

An equation relating internal member deformation "e" at any node to the external nodal displacements is given by

$$[e] = [B][X] \tag{2.21b}$$

where, both e and X may be rotations or translations. It can be shown using reciprocal theorem of structural mechanics that the [B] matrix is exactly the transpose of the [A] matrix, thus

$$[e] = [A]^T[X] \tag{2.21c}$$

The internal-member forces [F] are related to the internal member displacements as

$$[F] = [S]\,[e] \tag{2.21d}$$

Equations (2.21a), (2.21c) and (2.21d) are the fundamental equations in the finite element method of analysis.

Substituting (2.21c) into (2.21d)

$$[F] = [S][e] = [S][A]^T[X] \tag{2.21e}$$

And substituting (2.21e) into (2.21a)

$$[P] = [A][F] = [A][S][A]^T[X] \tag{2.21f}$$

Now the only unknown in this system of equations are the X's , so the $[A][S][A]^T$ is inverted to obtain

$$[X] = [[A][S][A]^T]^{-1}[P] \tag{2.21g}$$

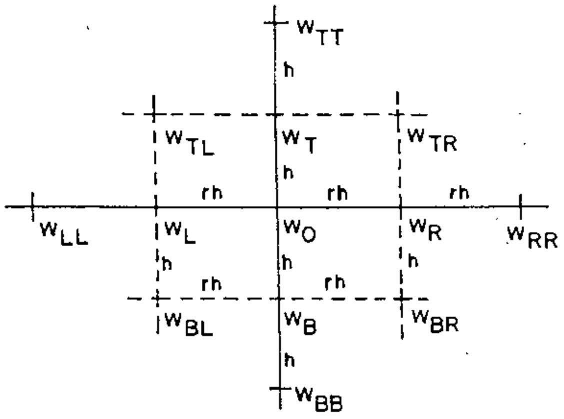

Fig. 2.3 Finite difference grid of elements of rh x h dimensions

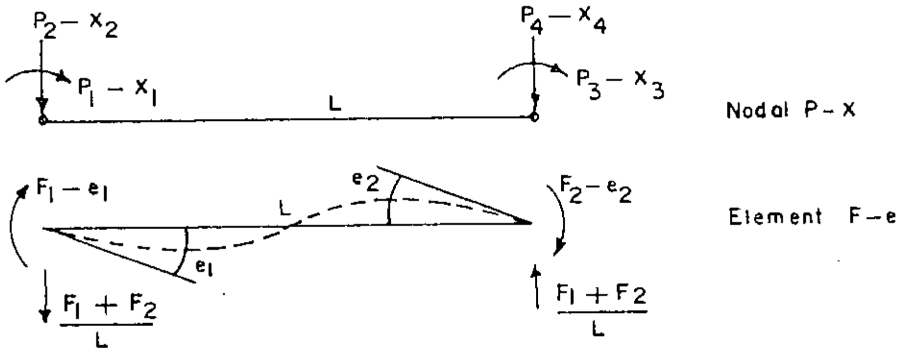

Fig. 2.4 External (nodal) and internal (member) finite element forces.

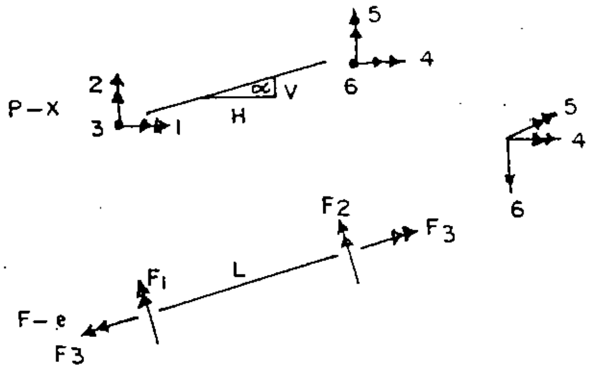

Fig. 2.5 P-X and F-e coding

and with the X's values known, one can back substitute into equation (2.21e) to obtain the internal-member forces which is necessary for design.

The $[A][S][A]^T$ matrix is often called the global matrix, since it represents the system of equations for each P and X node entry. It is convenient to build it from one finite element of the structure at a time and use superposition to build the global $[A][S][A]^T$ from the element $[A][S][A]^T$ matrix.

2.5.2 Formation of [A] and [S] Matrices

Referring to Fig.2.5, the element [A] matrix is built by ΣF at each node. For example, at node 1,

$$P_1 = -F_1 \sin\alpha + 0.F_2 - F_3 \cos\alpha \tag{2.22a}$$

$$P_2 = F_1 \cos\alpha + 0.F_2 - F_3 \sin\alpha \tag{2.22b}$$

$$P_3 = \frac{F_1}{L} \quad + \frac{F_2}{L} \quad + 0.F_3 \tag{2.22c}$$

and the resulting [A] matrix is

$$[A]= \begin{bmatrix} -\sin\alpha & 0 & -\cos\alpha \\ \cos\alpha & 0 & -\sin\alpha \\ 1/L & 1/L & 0 \\ 0 & -\sin\alpha & \cos\alpha \\ 0 & \cos\alpha & \sin\alpha \\ -1/L & -1/L & 0 \end{bmatrix} \tag{2.23}$$

The [S] matrix for each element is

$$[S]=\begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} & 0 \\ \frac{2EI}{L} & \frac{4EI}{L} & 0 \\ 0 & 0 & \frac{\Omega GJ}{L} \end{bmatrix} \tag{2.24}$$

The upper left cornered 2x2 matrix is same for beam element. But for mat element, torsion is additionally included.

The torsional factor J should be computed for a rectangle. The adjust factor Ω is used to make the solution better fit the theoretical solutions as found in plate theory analysis. At present the Ω factor for a best fit is

$$\Omega = \frac{0.75L}{B} \leq 1.1 \tag{2.25}$$

where, L, B = grid element length and width .

2.5.3 Developing the Element $[A]^T$ and $[A][S][A]^T$ Matrices

The $[S][A]^T$ matrix is formed by multiplying the element [S] and the transpose of the element [A] matrix,

$$[A]^T=\begin{bmatrix} -\sin\alpha & \cos\alpha & 1/L & 0 & 0 & -1/L \\ 0 & 0 & 1/L & -\sin\alpha & \cos\alpha & -1/L \\ -\cos\alpha & -\sin\alpha & 0 & \cos\alpha & \sin\alpha & 0 \end{bmatrix} \tag{2.26}$$

$[S][A]^T=$

$$\begin{bmatrix} -\frac{4EI}{L}\sin\alpha & \frac{4EI}{L}\cos\alpha & \frac{6EI}{L^2} & -\frac{2EI}{L}\sin\alpha & \frac{2EI}{L}\cos\alpha & -\frac{6EI}{L^2} \\ -\frac{2EI}{L}\sin\alpha & \frac{2EI}{L}\cos\alpha & \frac{6EI}{L^2} & -\frac{4EI}{L}\sin\alpha & \frac{4EI}{L}\cos\alpha & -\frac{6EI}{L^2} \\ -\frac{\Omega GJ}{L}\cos\alpha & -\frac{\Omega GJ}{L}\sin\alpha & 0 & \frac{\Omega GJ}{L}\cos\alpha & \frac{\Omega GJ}{L}\sin\alpha & 0 \end{bmatrix} \qquad (2.27)$$

And, the element $[A][S][A]^T$ matrix after incorporating spring stiffnesses is given by,

$[A][S][A]^T =$

$\frac{4EI}{L}\sin^2\alpha$ $+\frac{\Omega GJ}{L}\cos^2\alpha$	$-\frac{4EI}{L}\sin\alpha\cos\alpha$ $+\frac{\Omega GJ}{L}\sin\alpha\cos\alpha$	$-\frac{6EI}{L^2}\sin\alpha$	$\frac{2EI}{L}\sin^2\alpha$ $-\frac{\Omega GJ}{L}\cos^2\alpha$	$-\frac{2EI}{L}\sin\alpha\cos\alpha$ $-\frac{\Omega GJ}{L}\sin\alpha\cos\alpha$	$\frac{6EI}{L^2}\sin\alpha$
$-\frac{4EI}{L}\sin\alpha\cos\alpha$ $+\frac{\Omega GJ}{L}\sin\alpha\cos\alpha$	$\frac{4EI}{L}\cos^2\alpha$ $+\frac{\Omega GJ}{L}\sin^2\alpha$	$\frac{6EI}{L^2}\cos\alpha$	$-\frac{2EI}{L}\sin\alpha\cos\alpha$ $-\frac{\Omega GJ}{L}\sin\alpha\cos\alpha$	$\frac{2EI}{L}\cos^2\alpha$ $-\frac{\Omega GJ}{L}\sin^2\alpha$	$-\frac{6EI}{L^2}\cos\alpha$
$-\frac{6EI}{L^2}\sin\alpha$	$\frac{6EI}{L^2}\cos\alpha$	$\frac{12EI}{L^3}+K_1$	$-\frac{6EI}{L^2}\sin\alpha$	$\frac{6EI}{L^2}\cos\alpha$	$-\frac{12EI}{L^3}$
$\frac{2EI}{L}\sin^2\alpha$ $-\frac{\Omega GJ}{L}\cos^2\alpha$	$-\frac{2EI}{L}\sin\alpha\cos\alpha$ $-\frac{\Omega GJ}{L}\sin\alpha\cos\alpha$	$-\frac{6EI}{L^2}\sin\alpha$	$\frac{4EI}{L}\sin^2\alpha$ $+\frac{\Omega GJ}{L}\cos^2\alpha$	$-\frac{4EI}{L}\sin\alpha\cos\alpha$ $+\frac{\Omega GJ}{L}\sin\alpha\cos\alpha$	$\frac{6EI}{L^2}\sin\alpha$
$-\frac{2EI}{L}\sin\alpha\cos\alpha$ $-\frac{\Omega GJ}{L}\sin\alpha\cos\alpha$	$\frac{2EI}{L}\cos^2\alpha$ $-\frac{\Omega GJ}{L}\sin^2\alpha$	$\frac{6EI}{L^2}\cos\alpha$	$-\frac{4EI}{L}\sin\alpha\cos\alpha$ $+\frac{\Omega GJ}{L}\sin\alpha\cos\alpha$	$\frac{4EI}{L}\cos^2\alpha$ $+\frac{\Omega GJ}{L}\sin^2\alpha$	$-\frac{6EI}{L^2}\cos\alpha$
$\frac{6EI}{L^2}\sin\alpha$	$-\frac{6EI}{L^2}\cos\alpha$	$-\frac{12EI}{L^3}$	$\frac{6EI}{L^2}\sin\alpha$	$-\frac{6EI}{L^2}\cos\alpha$	$\frac{12EI}{L^3}+K_2$

The element $[A][S][A]^T$ matrix is the stiffness matrix. As discussed earlier, soil media is considered as springs in this method; these springs should be included in the stiffness matrix. The K_1 and K_2 are two spring stiffnesses (or spring constants) which are inserted in the (3,3) and (6,6) position of the matrix. Thus in this approach the soil spring constants appear only in the diagonal term and when the mat at any point separates from the soil, the spring at that point is just made zero.

At the time of construction of element stiffness matrix, the global stiffness matrix for the whole mat-soil is built by properly placing element $[A][S][A]^T$ matrix components to the global matrix.

Once the global stiffness $[A][S][A]^T$ matrix is formed, the nodal displacements are found using, $[X]=[[A][S][A]^T]^{-1}[P]$. The element forces can then be found by $[F]=[S][A]^T[X]$.

2.6 MODULUS OF SUBGRADE REACTION

To calculate moment, shear and deflections in a mat foundation subjected to given loads, one has to determine the modulus of subgrade reaction (also called soil spring constant or coefficient of subgrade reaction). This modulus is a conceptual relationship between soil pressure and deflection and is defined as the ratio of the intensity of soil pressure to the soil deflection at a point of the surface of contact. It is expressed as,

$$k_s = \frac{q}{\delta} \tag{2.29}$$

where,

k_s = modulus of subgrade reaction in force/length³ units FL^{-3}

q = soil pressure in force/length² units FL^{-2}

δ = deflection in soil at q pressure intensity, in length units L

A typical plot of q versus δ from load tests is qualitatively shown in Fig.2.6a. If this type of curve is used to obtain k_s, which is the slope in equation (2.29), it is evident that the value depends on whether it is a tangent or secant modulus and location of the co-ordinates of q and δ. Obviously one could divide the q-δ curve into several regions so that k_s takes different values of the slope in the different regions. However, this tends to incorporate too much refinement into the problem since most analyses proceed on the basis of estimated or at best of approximate load test.

Fig.2.6b is the idealised stress-deformation curve in which k_s is constant upto a deflection X_{max} . Beyond X_{max} the soil pressure is a constant value defined by,

$$q_{con} = k_s(X_{max}) \tag{2.30}$$

The value of X_{max} may be directly estimated at some small value (say 12 to 25 mm) or from inspection of load settlement curve. It might also be estimated from the peak strain in a triaxial compression test. From the peak strain ε_{max} can be estimated,

$$X_{max} = \varepsilon_{max}\,(1.5 \text{ to } 2B) \tag{2.31}$$

2.6.1 Methods to Determine k_s

The most popular method to determine modulus of subgrade reaction is to use the plate load test shown in Fig.2.7.

Field load-test procedures uses square plates commonly 12x12 inch and round plates from 12 inch to 30 inch in increments of 6 inch. The plate thickness is commonly 1 inch to reduce bending, although the plates are machined so that the smaller plates can be stacked on the larger ones to reduce bending.

Field load testing requires some means of providing a large load to the plate. This can be accomplished by jacking against a crawler tractor or loaded flat-bed equipment trailers or driving tension piles to be used with a cross beam to provide the load reaction.

From the test result, a q-δ curve has to be drawn. Taking a suitable co-ordinate, one can find the slope of the curve, i.e. the modulus of subgrade reaction.

Since the curve is seldom straight over any appreciable range of deformation, one must arbitrarily choose co-ordinates. The Road Research Laboratory (1952) uses the pressure corresponding to 0.13 cm (.05 in), obtaining

$$k_s = \frac{q}{0.13} \quad \text{kg/cm}^3 \tag{2.32}$$

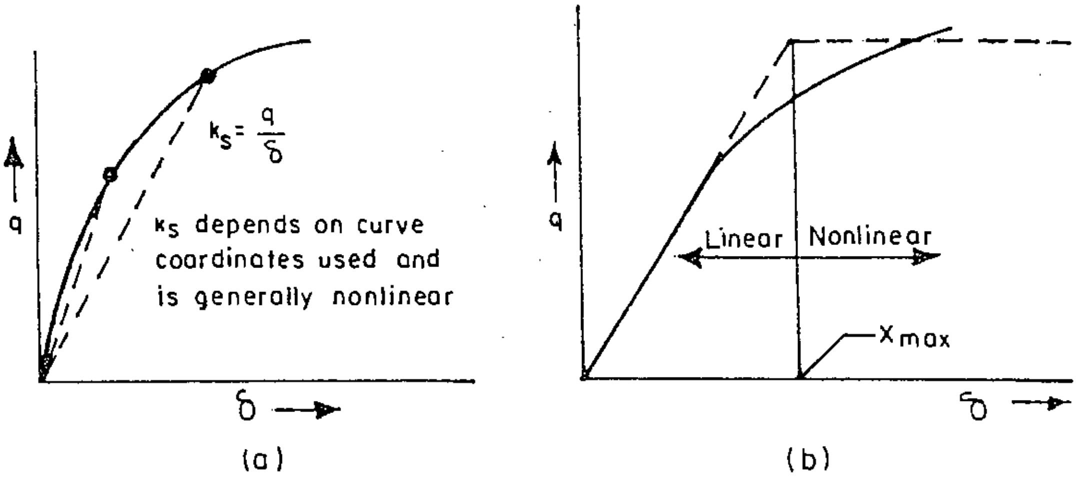

Fig. 2.6 Detemination of modulus of subgrade reaction from q–δ curve.

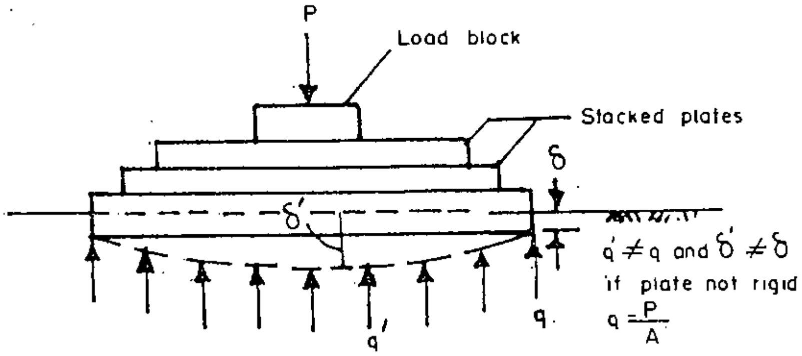

Fig. 2.7 Determination of k_s using plate load test.

The U.S corps of Engineers has used the deformation corresponding to 0.7 kg/sq. cm (10 psi) for evaluation of subgrade for airfields as

$$k_s = \frac{0.7}{\delta} \quad \text{kg/cm}^3 \tag{2.33}$$

The Navdocks Design Manual (1961) uses the pressure and corresponding deflection at one-half the yield pressure. The data can be plotted to a log-log scale, and where tangents of two straight-line parts of the resulting curve intersect, can be taken as yield pressure.

California Bearing (CBR) tests have also been used to obtain the subgrade modulus. The penetration load at 0.25 cm (0.1 in) converted to pressure (load per area, where area is computed based on a piston diameter of 4.95 cm) yields

$$k_s = \frac{q_{CBR}}{0.25} \tag{2.34}$$

A laboratory method proposed by Vesic' (1961,1963) uses the stress-strain modulus obtained from triaxial tests.

The Vesic' equation for subgrade reaction is

$$k'_s = 0.65 \sqrt[12]{\frac{E_s B^4}{E_b I_b}} \cdot \frac{E_s}{1-\mu^2} \tag{2.35}$$

where,

$k'_s = k_s B$,(FL^{-2})
Es = Stress-strain modulus (FL^{-2})
μ = Poisson's ratio
B = Width of footing
$E_b I_b$= Flexural rigidity for footing

Inspection of equation (2.35) indicates that $0.65 \sqrt[12]{\frac{E_s B^4}{E_b I_b}}$ is approximately equal 0.9 to 1.5, with an average value for footing of modest proportion of about 1.2; thus

$$k'_s \approx 1.2 \frac{E_s}{1-\mu^2} \tag{2.36}$$

and is accurate to the degree of precision of laboratory determination of E_s. Bowles (1974) pointed out that Vesic' values of k_s are lower and this is due to the lower values of E_s found in laboratory tests and suggested that the Vesic' equation is satisfactory if one can obtain the correct value of E_s.

2.6.2 Modification of k_s Values

Shukla (1984) outlined the following procedure to determine the value of k_s from k_{v1}, subgrade modulus using 1 ft^2 plate, based on Terzaghi's recommendation:

(1) Overall thickness t of mat is calculated from shear requirements, at critical section (as per ACI 318-83 section 11.11).

(2) Modulus of elasticity E_c of concrete is determined using the formula $E_c=57000\sqrt{f'_c}$ and assuming a suitable Poisson's ratio μ between 0.15 and 0.25 (higher value as concrete ages).

(3) Flexural rigidity D of the mat section is calculated from equation (2.7).

(4) Trial value of k_s is calculated as in the case of beams

a) for clay

$$k_s = \frac{k_{v1}}{B_c} \tag{2.37}$$

b) for sand

$$k_s = k_{v1}\left[\frac{B_c+1}{2B_c}\right]^2 \tag{2.38}$$

where,

k_s = modulus of subgrade reaction, kip/ft^3

B_c = spacing of column or line load due to walls, ft

k_{v1} = Modulus of subgrade reaction based on average results obtained by applying load through a 1 ft^2 bearing plate.

For the case where the spacings of column lines in two orthogonal directions are different, the average of the two spacings should be taken for B_c.

(5) The radius of effective stiffness is calculated from equation (2.8). The distance 4L is called radius of influence because the responses due to a given load dampen quickly after this distance.

(6) The revised value of k_s may be calculated, if required, as follows:

a) When the column spacing in either direction, exceeds 4L, B_c= 4L is used in equation (2.37) or (2.38)

b) When column spacings in both directions are smaller than 4L, a contact area equivalent to length B_{c1} and width B_{c2} is assumed and the formula for a rectangular plate is used as

$$k_s = k_{v1} \frac{m+0.5}{1.5m} \tag{2.39}$$

where m = B_{c1}/B_{c2} , B_{c1} and B_{c2} are column spacings in orthogonal directions.

Shukla (1984) also presents a chart (Fig.2.8) giving values of modulus of subgrade reaction based on average results obtained by applying load through a 1 ft^2 bearing plate. Using this chart one can find k_{v1} if unconfined compressive strength q_u, of clay soil or relative density of sandy soil is known.

2.6.3 Approximate Value of k_s

Bowles (1988) suggested the following equations for approximating k_s from the allowable bearing capacity,

for FPS : $k_s = 12(SF)q_a$ k/ft^3 (2.40a)
for SI : $k_s = 40(SF)q_a$ kN/m^3 (2.40b)

where,

q_a = Allowable bearing capacity in ksf or kPa
SF= Safety factor

These equations are based on $q_a = q_{ult}/SF$ and the ultimate soil pressure is at a settlement ΔH= 1 in. (1/12 ft) or 0.0254 m and k_s is $q_{ult}/\Delta H$.

Bowles (1988) also presented a table (Table 2.1) showing range of values of k_s for different types of soil. This table (2.1) may be used to estimate a value of k_s to determine the correct order of magnitude of the modulus of subgrade reaction obtained using one of the above approximate equations.

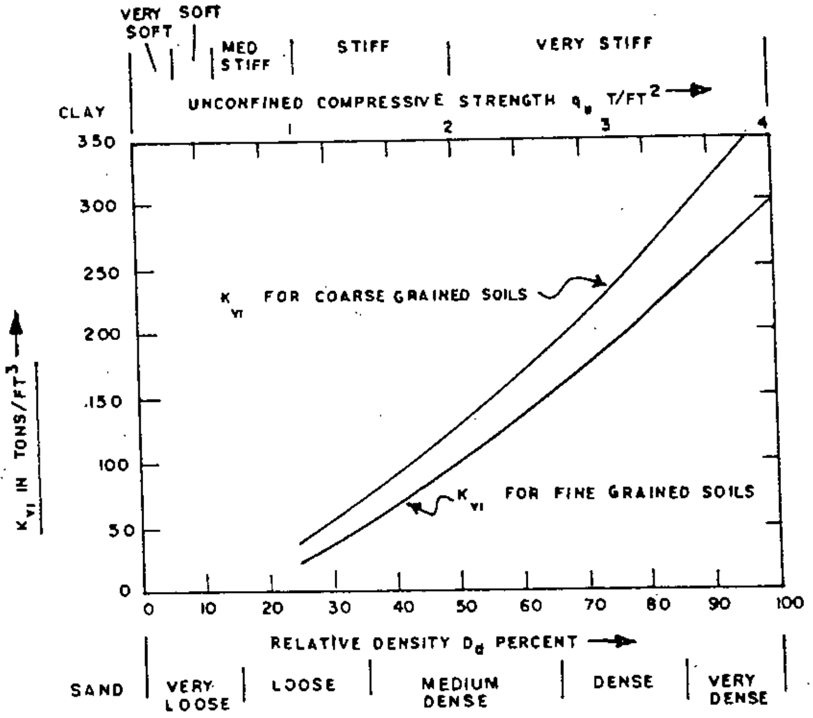

Fig. 2.8 Modulus of subgrade reaction k_s of soil after Shukla (1984)

Soil	k_s, kcf	k_s, kN/m3
Loose sand	30 - 100	4,800 - 16,000
Medium dense sand	60 - 500	9,600 - 80,000
Dense sand	400 - 800	64,000 - 128,000
Clayey medium dense sand	200 - 500	32,000 - 80,000
Silty medium dense sand	150 - 300	24,000 - 48,000
Clayey soil :		
$q_u \leq 200$ kPa(4 ksf)	75 - 150	12,000 - 24,000
$200 < q_u \leq 400$ kPa	150 - 300	24,000 - 48,000
$q_u > 800$ kPa	> 300	> 48,000

Table 2.1- Ranges of values of modulus of subgrade reaction
[after Bowles (1988)]

CHAPTER 3

THE COMPUTER PROGRAMS

3.1 INTRODUCTION

Different methods to analyse a mat foundation are presented in details in the preceding chapter. The purpose of this chapter is to employ available computer programs after incorporating suitable modifications to them and also to develop new programs for those methods for which programs are not readily available.

The program using finite difference method is from Bowles (1974). The program has been suitably adapted for microcomputers and present-day compilers. Finite grid method program is taken from Bowles (1988) after modification. A data generation program is also employed for this program. A computer program is developed for the ACI approximate flexible method. All these programs are schematically described in flow charts presented in the appendix.

All the programs discussed here are in FORTRAN77 and were tested and run on the 486 microcomputer at the computer centre of Civil Engineering Department. Conventional method is not considered for developing program since it is suitable and convenient for hand calculation.

3.2 COMPUTER PROGRAM FOR FINITE DIFFERENCE METHOD

This program solves any square or rectangular mat using a square or rectangular grid. As discussed in the preceding chapter that for a given mat one difference equation is written for each point of intersection of the grid system. Thus in this method a large matrix of difference equations is formed and a major problem of this method is to solve this large matrix. In this program this solution is done by product inverse method. This method is slow, both because of very large number of computations and because the matrix is stored in temporary file and only one column at a time is operated on. Although alternative special equation solving techniques are available, the product-inverse method appears to be as rapid as any [Bowles (1974)].

3.2.1 Input Data for the Program

Input data for the finite difference program is very simple and small in number; no data generation program is required. It requires number of horizontal and vertical lines forming the grid and number of loads acting on this grid intersecting points. The horizontal and vertical grid size, thickness of mat, vertical and horizontal co-ordinates of points where loads are acting and magnitude of loads are necessary. The only soil property required is the modulus of subgrade reaction and as for concrete, modulus of elasticity, unit weight and Poisson's ratio are needed.

3.2.2 Output of the Program

The output of the program gives deflections of mat at every intersection node point of grid lines. The bending moments per unit length of the plate in both directions are also provided at every node. The nodal soil pressure and soil reaction i.e. spring forces at every node are presented in the output. Finally the program provides total sum of external load along with footing weight and the total soil reaction to check whether vertical force equilibrium is maintained.

3.3 COMPUTER PROGRAM FOR FINITE GRID METHOD

This programs can analyse any square or rectangular mat using square or rectangular grid, like finite difference program. Unlike the finite difference method, the grids in any direction can have different sizes. Here, as discussed in previous chapter, stiffness of each element along with the two end soil springs is developed one by one and its position in the global stiffness matrix is found. Thus one can use different values of soil springs to take into account the dishing effect of mat which is discussed in the next articles.

3.3.1 Input Data for the Program

The input data for the program is enormous. To describe the mat configuration it needs mat thickness, total number of variables (three being at each node), total number of beam-column elements, bandwidth of the stiffness matrix, number of loads at nodes, magnitude of loads and variable numbers corresponding to these loads. The large part of the input file comprises of the member incedences (i.e. for each member six variables at two ends). The input also needs horizontal and vertical lengths and widths of the elements. The soil parameters needed are modulus of subgrade reaction and maximum linear soil deflection. As for concrete, the modulus of elasticity, shear modulus, unit weight are needed. The program can calculate soil springs acting at each node from modulus of subgrade reaction of soil. The spring values can also be directly given to the program as input.

3.3.2 Data Generation

The finite grid program requires enormous input data which is difficult to write correctly by hand calculations. For this reason, a data generation program is developed which requires only a few input data concerning the mat and creates the input file for the finite grid program. This generation program also incorporates a subroutine to determine the vertical stress profile using Newmark's (1935) method which is used to zone the soil spring as discussed in the next article. Thus softer soil springs are created towards the centre of the mat than the edges.

The data generation program reads the number of horizontal and vertical grids of the mat and their sizes. From these values it creates number of nodes, number of variables, number of members and band width of the stiffness matrix. It also generates member number, six variable numbers per element, horizontal, vertical projection and width of each element. To create the load matrix, it generates the non zero elements of the matrix by reading only the position and magnitude of loads.

3.3.3 Soil Spring Zoning

All the discrete element methods discussed in the previous chapter use modulus of subgrade reaction k_S to support the plate. The subgrade reaction is used to compute node springs based on the contributing areas. In this form the springs are independent of each other and the springs are called uncoupled; uncoupling means that the deflection of any spring is not influenced by adjacent springs.

If the springs are to be coupled, simple methods are desired so that only the diagonal term of the stiffness matrix is affected. This allows for ease in adjusting for non-linear effects.

Bowles (1986) suggested that coupling can be done as follows:

(1) The edge springs of a mat are simply doubled in magnitude.
(2) The mat area can be zoned using softer springs at the centre than the edges and using intermediate zones to transition from the edge to the centre.
(3) Both 1 and 2 above can be employed at a time for the extreme case.

The second method is described as follows:

(1) Vertical stress profiles at selected points beneath the foundation are obtained using Newmark (1935) method for vertical stresses . A depth of 4B to 5B is generally used. The equation given by Newmark which is applicable beneath the corner of a rectangular area is :

$$q_v = q_0 \frac{1}{4\pi}\left[\frac{2MN\sqrt{V}}{V+V_1}\frac{V+1}{V} + \tan^{-1}\left(\frac{2MN\sqrt{V}}{V-V_1}\right)\right] \tag{3.1}$$

where,

q_v = Vertical pressure at Z depth
q_0 = Pressure at surface
L = Length of footing
B = Breadth of footing
M = B/Z
N = L/Z
V = M^2+N^2+1
V_1 = $(MN)^2$

(2) The vertical stress profile at a point is numerically integrated to obtain the average stress increase DQ.

(3) Modified values of k'_{si} for any point i is computed as

$$k'_{si} = k_s \frac{DQ_e}{DQ_i} \tag{3.2}$$

where,

DQ_e = Average stress increase for a point on the edge of a square plate
DQ_i = Average stress increase at any point i.

This method is employed in the data generation program of the finite grid method which has been discussed in the previous article.

3.3.4 Output of the Program

The program first gives the nodal soil spring array which is either calculated from soil modulus or directly supplied to the program as input. It also gives the load matrix formed and the deflections of each node calculated by the program. For each element, bending moments at two ends, torsional moment, shear are presented. The calculated nodal bending moments in both the directions, spring reaction and nodal soil pressure at all the nodes are also provided in the output. The output also supplies total applied forces and total soil reaction to check whether vertical force equilibrium is maintained.

3.4 COMPUTER PROGRAM FOR ACI APPROXIMATE FLEXIBLE METHOD

In the previous chapter, it was discussed that this method was proposed by ACI Committee 336 (1966). In their conclusion the committee suggested that the use of electronic computers should be considered in using this method. Although Bowles (1988) recommends this method only as a spot check of other solutions because it involves large computations which may cause errors, this method may prove suitable if a computer program can be developed to take care of enormous computations. Mician (1985) also ascertained the proposal of ACI Committee 336 and himself developed a program and presented results in his paper using this program. A program is developed in FORTRAN77 employing this method and is discussed in the following articles.

This method is based on the theory of infinite plates where the plate is extended to infinity in both directions. In reality the mat is a finite structure and corrections should be made for the end conditions of mat. Shukla (1984) uses a method to take into consideration of these end conditioning forces which is discussed in the previous chapter. Here, end conditioning forces are applied to each unit width strip of mat and as each strip is considered independently of others, the summation of vertical forces does not act as a check as in other methods. Thus to check whether the program is correctly coded, a number of trial cases were run where the mat is made thin with very stiff soil so that the mat can be idealised as infinite. It has been found that the total soil pressure agrees with the total column loads within 10 percent variation (shown in appendix B). Shukla (1984) presented moment-, shear- and deflection-coefficients for an infinite mat on elastic foundation using Z functions (discussed in article 2.3.2) and presented in Fig.2.2. The same coefficients are calculated in the present computer program using basic Z functions.

3.4.1 Input Data for the Program

Unlike finite difference and finite grid methods, ACI method needs no gridding and only a small number of data is required for the analysis. To define the mat problem, one has to supply length and breadth of the mat and the number of columns. Co-ordinates of columns along with their sizes and loads should be specified. The only soil parameter needed is the modulus of subgrade reaction and for concrete, Poisson's ratio and strength should be given. The co-ordinates of points at which moment, shear and deflections are to be found should be given as input, as well.

3.4.2 Output of the Program

The program gives thickness of mat calculated on the basis of safety against punching and allowing clear cover for reinforcements. The output gives moments, shears and deflections at points whose co-ordinates are specified in the input. In an effort to enable the user to ascertain the effect of end conditioning forces on the solution (which assumes the mat to be infinite), the output contains results both before and after applying end conditioning forces.

CHAPTER 4

ANALYSIS SCHEME

4.1 INTRODUCTION

The method of analysing a mat-soil system has been described in the preceding chapters. The computer programs developed and modified in this study can now be used for analysing a variety of mat-soil systems. The analysis scheme is divided into two parts, firstly a trial mat is selected to be analysed by different methods to compare the results obtained by these methods, secondly several parameters are selected to determine their effects on moments, deflections at some selected point on the mat.

4.2 COMPARISON BETWEEN DIFFERENT METHODS

The methods to be compared are conventional method, ACI approximate flexible method, finite difference method and finite grid method. Also finite grid method with and without soil spring zoning should be compared. The results that are to be compared are moments and deflections along the column lines in both directions. Understandably, first step in the comparison is to idealise the problem.

4.2.1 Problem Idealisation

To carry out the comparative study, a building supported on mat is selected. Fig.4.1 shows the plan of the mat. The mat is 72 ft by 60 ft in plan dimensions. The columns are spaced equally 16 ft apart in the x-direction and in the y-direction they are spaced 12 ft, 20 ft and 20 ft apart. In all the directions, the mat is extended 4 ft from the centre lines of the exterior columns. The building is 7 storied having each storey 12 ft high. All the columns are 20 inch by 20 inch in size and the mat is 2.5 ft thick which is safe against punching due to column loads described in the next section.

4.2.2 Load Calculation

The loads on mat from columns include dead load, live load and wind load. Dead and live loads are determined on the basis of contributing area of slab and the slab is assumed to be a flat plate having 9 inch thickness. Dead load from slabs and columns are determined on the basis of concrete unit weight 150lb/cft. The dead load of the mat itself is excluded for two reasons; firstly as concrete is poured on the soil directly, it does not cause any bending and secondly as concrete replaces the soil having the same order of unit weight, contribution of the unit weight of mat in deflection is similar to the contribution of the soil mass removed. The live load used in the problem is 80 psf according to Winter (1981) which allows 20 psf for partition wall. As the full live load on all the spans is never applied in reality, a reduction in live load as per Winter (1981) is used. It results in 60% reduction in the live load.

To determine the wind load, Standard Building Code given in Taranath (1988) is used which calculates pressure on the building surface using the equation:

$$p_s = 0.0025\, v^2 \left(\frac{H_b}{30}\right)^{\frac{2}{7}} \tag{4.1}$$

where,

p_s = Pressure on surface, psf
v = Velocity of maximum probable wind, mph
H_b= Height of the point where pressure is to be calculated, ft.

It has been found that for a storm of 120 mph on a building of 84 ft height, the pressure is 0.05 ksf. Pressure at different levels are determined and the force on the wall is calculated. To find the effect of wind on mat, the column loads are increased in one side and decreased on the other according to Mician (1985). It has been found that for a 84 ft building, the loads due to wind is very small compared to dead and live loads. As given in Winter (1981) the total factored design load is given by

$$U = 1.4DL + 1.7LL \tag{4.2}$$

and if wind is included

$$U = 0.75\ (1.4DL+1.7LL+1.7WL) \tag{4.3}$$

Clearly in this particular case equation (4.2) governs. The design load on each column is shown on the Fig. 4.1.

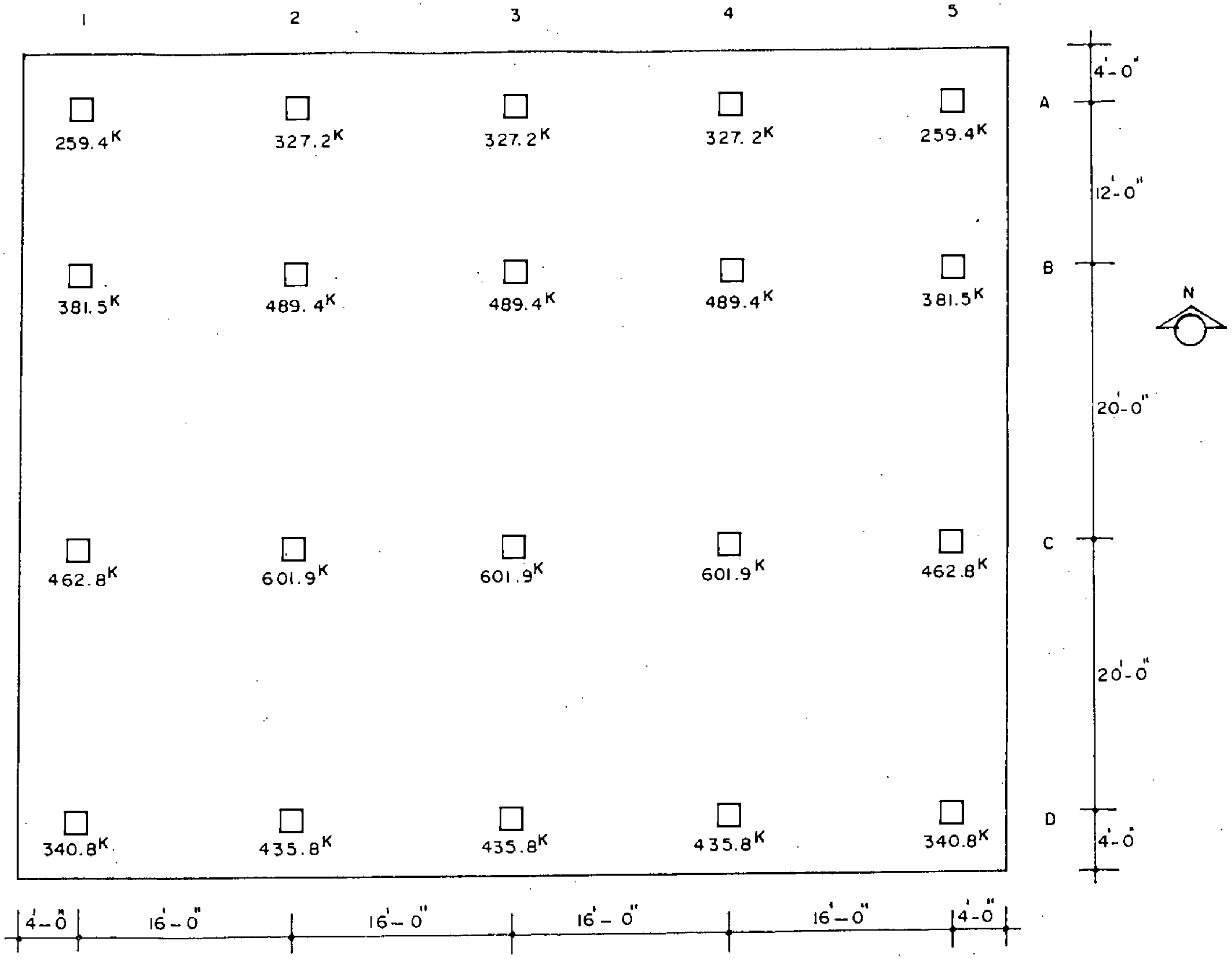

Fig. 4.1 Mat dimensions with column loads.

4.2.3 Material Properties

The most important soil property required in the analysis is the modulus of subgrade reaction (k_s). A value of 100 kcf is taken and this value is assumed on the basis of equation (2.40a) with safety factor 3 and for soil having allowable bearing capacity $q_a = 3$ ksf which is reasonable for Dhaka city. The other soil parameter required is the maximum linear soil deflection which is assumed to be 0.75 inch according to Bowles (1988).

The properties of (mat) concrete required are concrete strength f'_c, unit weight and Poisson's ratio (μ). The concrete strength is assumed to be 3000 psi, unit weight to be 150 lb/cft and Poisson's ratio to be 0.15 . Modulus of elasticity of concrete is calculated using $E_c = 57000 \sqrt{f'_c}$ psi. Poisson's ratio 0.15 represents the early life stage of concrete.

4.2.4 Items to be Compared

To compare the performance of different methods described earlier, certain items are selected which are important for a mat design. These are moments in both directions and deflections at column points. Moments are compared along column line A, B, C, D, 1, 2 and 3 as shown in Fig. 4.1. Deflections are to be calculated also along column line A, B, C, D, 1, 2 and 3.

4.3 PARAMETRIC STUDY

The objective of parametric study is to observe the effect of several parameters involved in the behaviour of the mat-soil system. The parameters selected are modulus of subgrade reaction, concrete strength, Poisson's ratio and mat thickness.

These parameters are varied over a reasonable range to find the response in mat-soil system. Moments in both directions and deflections at some selected points on the mat are compared to observe this response.

4.3.1 Effect of Modulus of Subgrade Reaction

To observe the effect of modulus of subgrade reaction, the mat problem used in comparative study of different methods as described in article 4.2 is selected.

Now the problem is modified for different subgrade modulus values keeping all other parameters constant. The modulus values selected for the study are 50, 100, 200 and 300 ksf.

4.3.2 Effect of Stiffness of Mat

The same mat problem described in article 4.2 is selected to find the effect of stiffness of mat. In order to monitor its effect, both concrete strength and mat thickness are varied independently keeping all other parameters unchanged. Here mat thickness have been varied from 30 to 48 inch in step of 6 inch. The concrete strengths selected for the study are 2500, 3000, 4000 and 5000 psi. It may be mentioned here that no reduction in mat thickness has been performed (although a higher strength may allow such a reduction) to study the effect of the parameter under consideration independently.

4.3.3 Effect of Poisson's Ratio of Concrete

To determine the effect of Poisson's ratio of concrete, the same mat problem described in article 4.2 is selected. Here Poisson's ratio is varied keeping all other parameters constant. Poisson's ratio selected for the study are 0.15, 0.18, 0.22 and 0.25. The values of Poisson's ratio generally lies between 0.15 and 0.25 . A value of 0.15 represents an early aged concrete. The value is dependent on age of concrete and increases with time.

4.3.4 Effect of Load

To determine the effect of vertical and horizontal loads, the same plan dimensions described in 4.2.1 are used, but the building is extended to 14 stories. The dead and live loads are calculated for each column and design column loads are established. It has been found that doubling the number of stories result in doubling the design load for each column. The behaviour of mat due to doubling the loads are to be considered, i.e. the moments and deflections for a 14 and 7 storied building are to be compared if the plan remain identical.

To determine the effect of wind load, a storm of 120 mph is considered both from north and south directions. Using equation (4.1), wind pressures on building are calculated at different levels and wind shear and moment are also determined. Using Mician's (1985) technique, upward and downward loads which are equivalent to the wind moment are calculated. But code [Winter (1981)] specifies that design load can not be less than that given by equation (4.2). Thus for a northward wind, loads are increased for column line A and B and for a southward wind, loads are increased for column line C and D. Bending moments and deflections in both the directions are to be compared for cases of wind from two opposite directions with the case when wind load is not present.

4.3.5 Effect of Number and Spacing of Columns

To find the effect of column number and spacing, the same mat-soil system used in the comparative study is considered. Two cases are studied; firstly the five columns along column line B are dropped and secondly eight columns along column line 2 and 4 are dropped. In both cases, the plan area with other parameters remains unchanged and new column loads are calculated. The effect of these dropping of columns on bending moments along column lines have been studied.

CHAPTER 5

DISCUSSIONS ON RESULTS OF THE COMPARATIVE STUDY

5.1 INTRODUCTION

In the previous chapters , the methods available for analysis of a mat, computer programs developed for them and the analysis scheme for the comparative study of these methods has been described. This chapter describes the results obtained by different methods using computer programs. For conventional method the soil pressure distribution under each strip and the modified column loads are calculated manually and the moments at an interval along the strip are calculated using a beam solving program.

There are two parts of the analysis work: firstly a comparative study is carried out to compare results obtained by different methods; secondly, the effects of different selected parameters on the behaviour of the mat-soil system are explored. Both approaches of the comparative study of different methods and the variational study of different parameters are outlined in the analysis scheme as reported in chapter 4.

5.2 SELECTION OF GRID SIZE

To analyse the mat-soil system by finite grid and finite difference methods, the mat needs to be divided into small elements by gridding. The gridding should be such that the elements are small enough to produce realistic results. To establish a reasonable grid size for the analyses, both 4 ft and 2 ft grids are employed for finite grid method with soil spring zoning. Moments along column line A and C obtained with 2 and 4 ft gridding are presented in Fig.5.1 and 5.2 respectively. As the mat in Fig.4.1 (described in chapter 4) is symmetrical along in the east-west direction, halves of symmetrical moment diagrams are presented. Both at column line A and C , positive moments (at column points) are higher for 2 ft grids than for 4 ft grids. At column A1, A2, A3, C1, C2, and C3, 2 ft grid results are 2.8%, 2.9%, 4.0%, 10.0%, 8.7% and 8.5% higher than respective 4 ft grid results. In both column lines, 4 ft grid yields higher negative moments between columns.

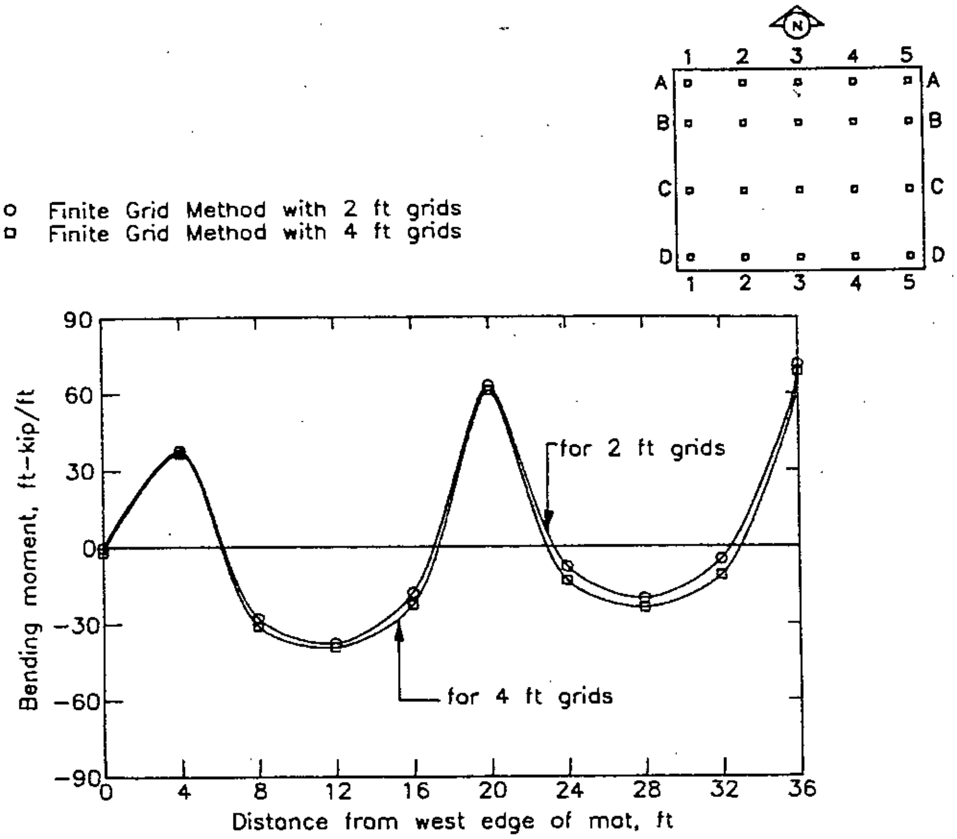

Fig.5.1 Variation of moments along column line A with 2 ft and 4 ft grids

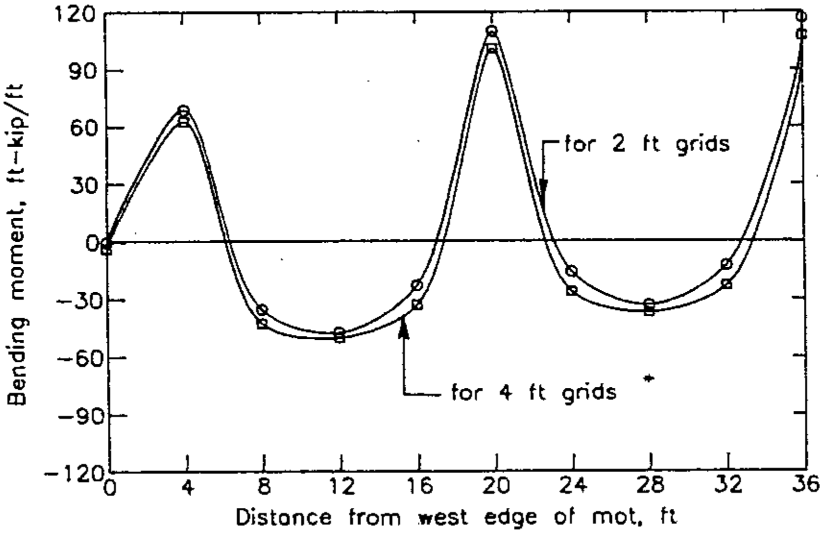

Fig.5.2 Variation of moments along column line C with 2 ft and 4 ft grids

The deflection along column line A and C obtained by 2 and 4 ft grids are presented in Fig.5.3 and 5.4. Here, 2 ft and 4 ft grids yield almost identical deflections for all the points. The maximum variation is at the west edge point; here 2 ft grid result is 2.6% higher than that of 4 ft grid.

Now, looking at the minute difference in deflection and slight difference in the computed values of moment due to 4 times finer grids, it can be easily inferred that, finer grid size does not readily ensure more realistic results. On the contrary, a decrease in the grid spacing results in a huge increase in the computational time and memory. Thus, from a practical point of view, considering the possible margin of error in numerical computation, and the freedom that a structural analyst can exercise, an uniform grid of 4 ft has been employed in all the analysis performed in this study. Due to computer time constraints, a smaller mat is analysed with 2 ft and 4 ft grids using finite difference method and results are presented in appendix C.

5.3 COMPARISON OF DIFFERENT METHODS

The moments at every column line in both directions of the mat has been compared. The column lines are column line A, B, C, D 1, 2 and 3. The mat is symmetrical in east-west direction and moments along column line 4 and 5 are identical to those along column line 2 and 1, respectively.

5.3.1 Moment along Column Lines

The variation of moment along column line A is presented in Fig.5.5. Here results obtained from five different approaches are shown. Column line A is the exterior column line along north side of the mat and there are five columns in a row namely A1, A2, A3, A4 and A5. The first point to be identified is that the column line A is symmetrical in east-west direction considering mat dimensions and loads. It is evident from the graph that the moment diagrams obtained from all the methods are symmetrical about column A3.

At column A3, the positive moment obtained by finite grid method with soil spring zoning is the highest of all methods and it is found to be 68.6 ft-kip/ft. The values obtained by finite grid method without soil spring zoning, finite difference method, and ACI approximate method are 63.13, 59.06 and 46.44 ft-kip/ft, respectively.

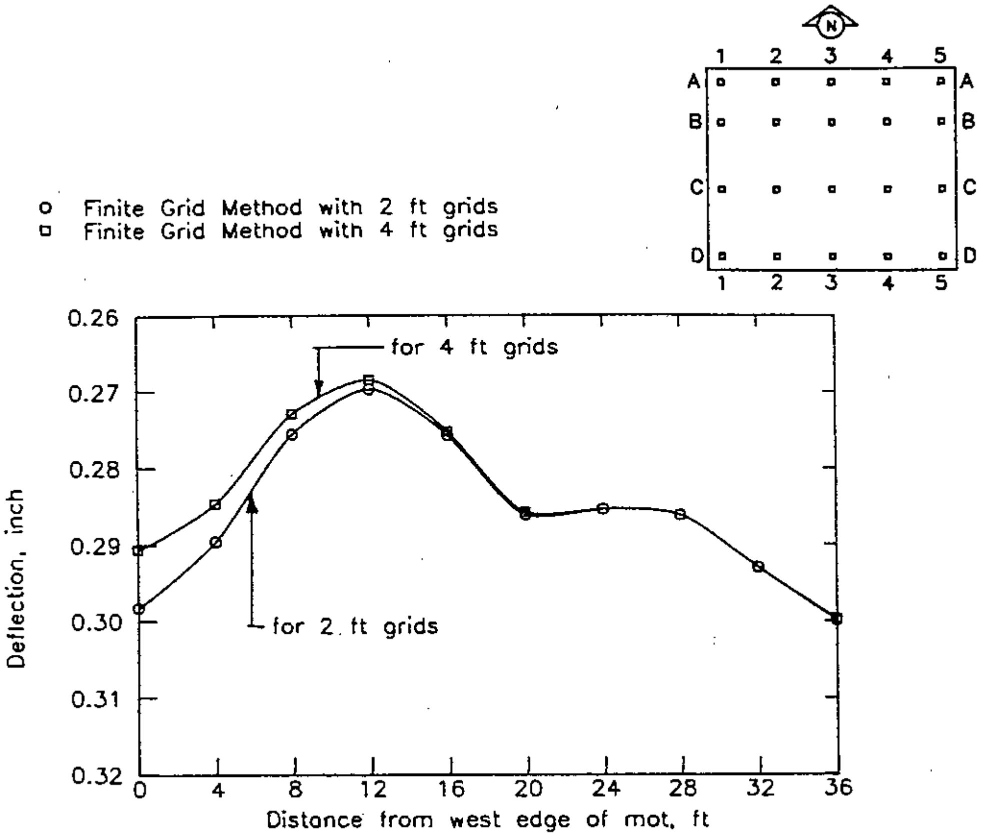

Fig.5.3 Variation of deflections along column line A with 2 ft and 4 ft grids

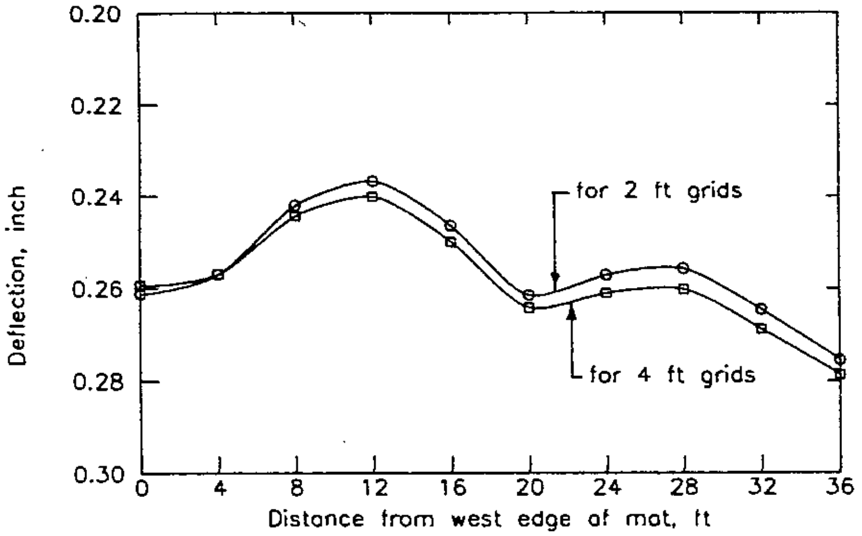

Fig.5.4 Variation of deflections along column line C with 2 ft and 4 ft grids

These values are smaller than the finite grid method with soil spring zoning by 8.0%, 13.9% and 32.3% respectively. The moment obtained by conventional method is -3.13 ft-kip/ft and it is not at all comparable to the results obtained by other methods. This may be due to two reasons, firstly the soil pressure distribution under the column strip is assumed to be constant, which creates larger negative moments between columns and lesser positive moments at column points; secondly here the mat is analysed strip by strip, the moment at a strip is divided by the strip width which is the summation of half bay length from each side of the column and as a result the high values of moment at column points are flattened over the entire strip.

The moments at column point A2 by finite grid method without soil spring zoning, finite difference method and ACI approximate flexible method are 56.53, 52.27 and 44.17 ft-kip/ft. The moment obtained by finite grid method with soil spring zoning is 61.13 ft-kip/ft. The values obtained by other methods are lesser than this value by 7.5%, 14.5% and 27.7% respectively. Here moment obtained by conventional method is 1.71 ft-kip/ft which is also very small in comparison to other methods.

At column A1, the moment obtained by finite grid method with soil spring zoning is still the highest of all methods and found to be 36.47 ft-kip/ft. The results obtained by finite grid method without zoning, finite difference method, conventional method and ACI approximate method are 35.97, 32.65, 16.33 and 6.44 ft-kip/ft. These moments are less than finite grid with zoning by 1.4%, 10.5%, 55.2% and 82.3%, respectively. The effect of soil spring zoning at this corner column is very low compare to interior columns. Here, conventional method gives moment almost half of the finite grid method moment. But ACI approximate flexible method gives very low values compared to other methods. The cause of low moment prediction by the ACI method, at the corner column, lies in the fact that in this method, the moments are calculated considering infinite mat in all directions, and then end conditioning forces are applied taking into account 1 ft strip which results in very concentrated effects on ends of strips.

Conventional method gives higher (negative) moment values among all the methods at points between columns. Finite grid method without soil spring zoning gives higher values than other comparable method. The value of moment at the mid point between column A2 and A3 obtained by finite grid

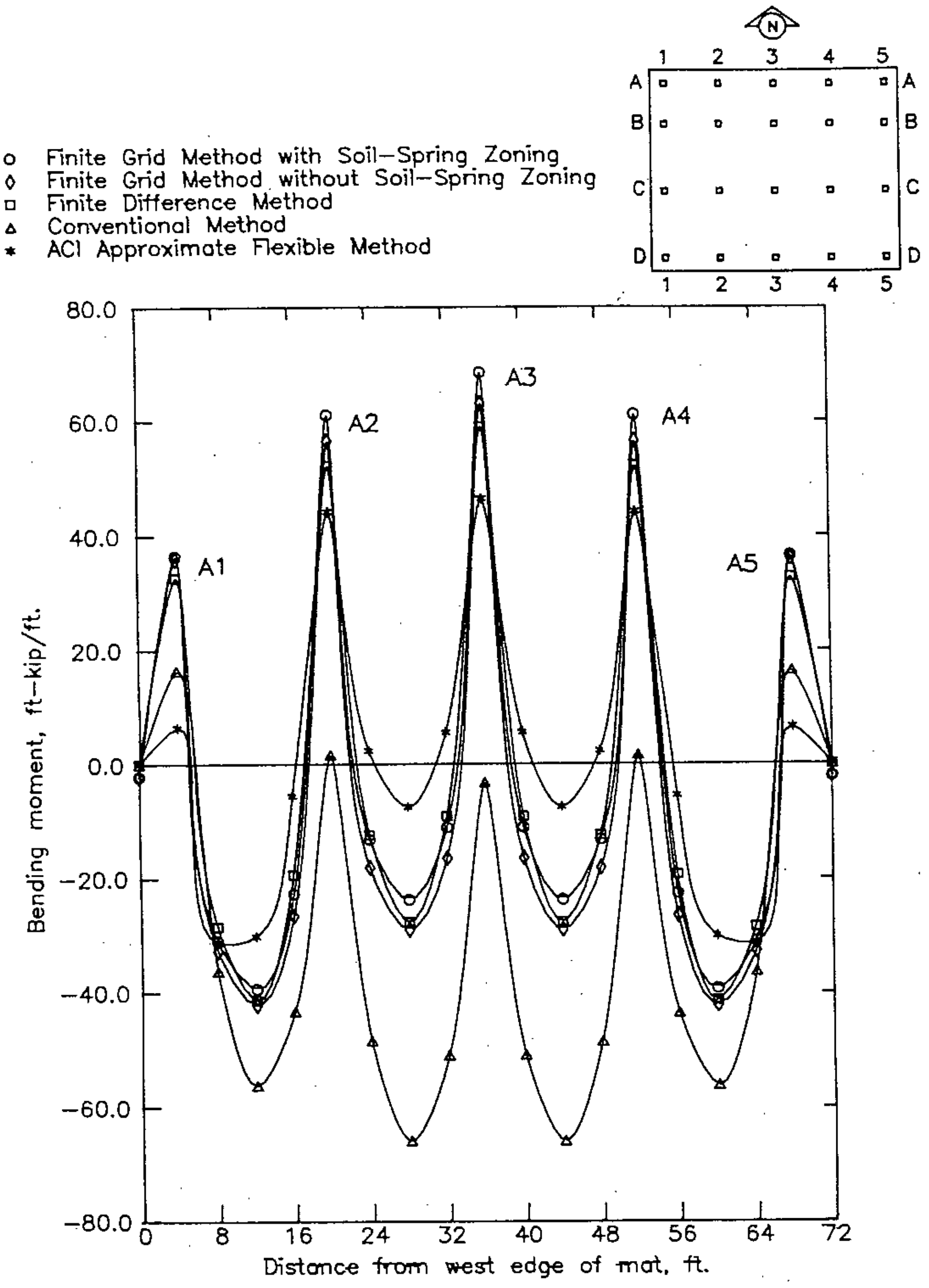

Fig.5.5 Comparison of moments along column line A

method with soil spring zoning is -23.85 ft-kip/ft. The moments obtained by finite grid method without zoning and finite difference method are -29.09 and -27.80 ft-kip/ft. These values are higher than finite grid method with soil spring zoning results by 22.0% and 16.6%. The moment by ACI method is -7.53 ft-kip/ft and it is less than finite grid method with zoning by 68.4%.

At a point halfway between column A1 and A2, the moment obtained by finite grid method with zoning is -39.44 ft-kip/ft. The results obtained by finite grid method without zoning, finite difference method and conventional method are -42.32, -41.44 and -56.31 ft-kip/ft and they are higher than their finite grid method with soil spring zoning counterpart by 7.3%, 5.1%, 42.8%. The ACI method still gives less values and at this point, the moment is -30.21 ft-kip/ft, which is 23.4% less than finite grid method with soil spring zoning. It has been found that finite grid method with soil zoning gives highest moment values at column points but between column points it gives values less than finite difference and finite grid method without soil spring. The moments obtained by different methods at points along column line A with their percentage variation are presented in Table 5.1.

Point	Bending moments, ft-kip/ft			
	Finite grid method with soil spring zoning	Finite grid method with out soil spring zoning	Finite difference method	ACI approximate flexible method
A1	36.47 (100.0 %)	35.97 (98.6 %)	32.65 (89.5 %)	6.44 (17.6 %)
A2	61.13 (100.0 %)	56.53 (92.5 %)	52.27 (85.5 %)	44.17 (72.3 %)
A3	68.60 (100.0 %)	63.13 (92.0 %)	59.06 (86.1 %)	46.44 (67.7 %)
A1-A2	-39.44 (100.0 %)	-42.32 (107.3 %)	-41.44 (105.1 %)	-30.21 (76.5 %)
A2-A3	-23.85 (100.0 %)	-29.09 (121.9 %)	-27.80 (116.6 %)	-7.53 (31.6 %)

Table 5.1 : Comparison of moments along column line A

The moment diagram along column line B is presented in Fig.5.6. At column

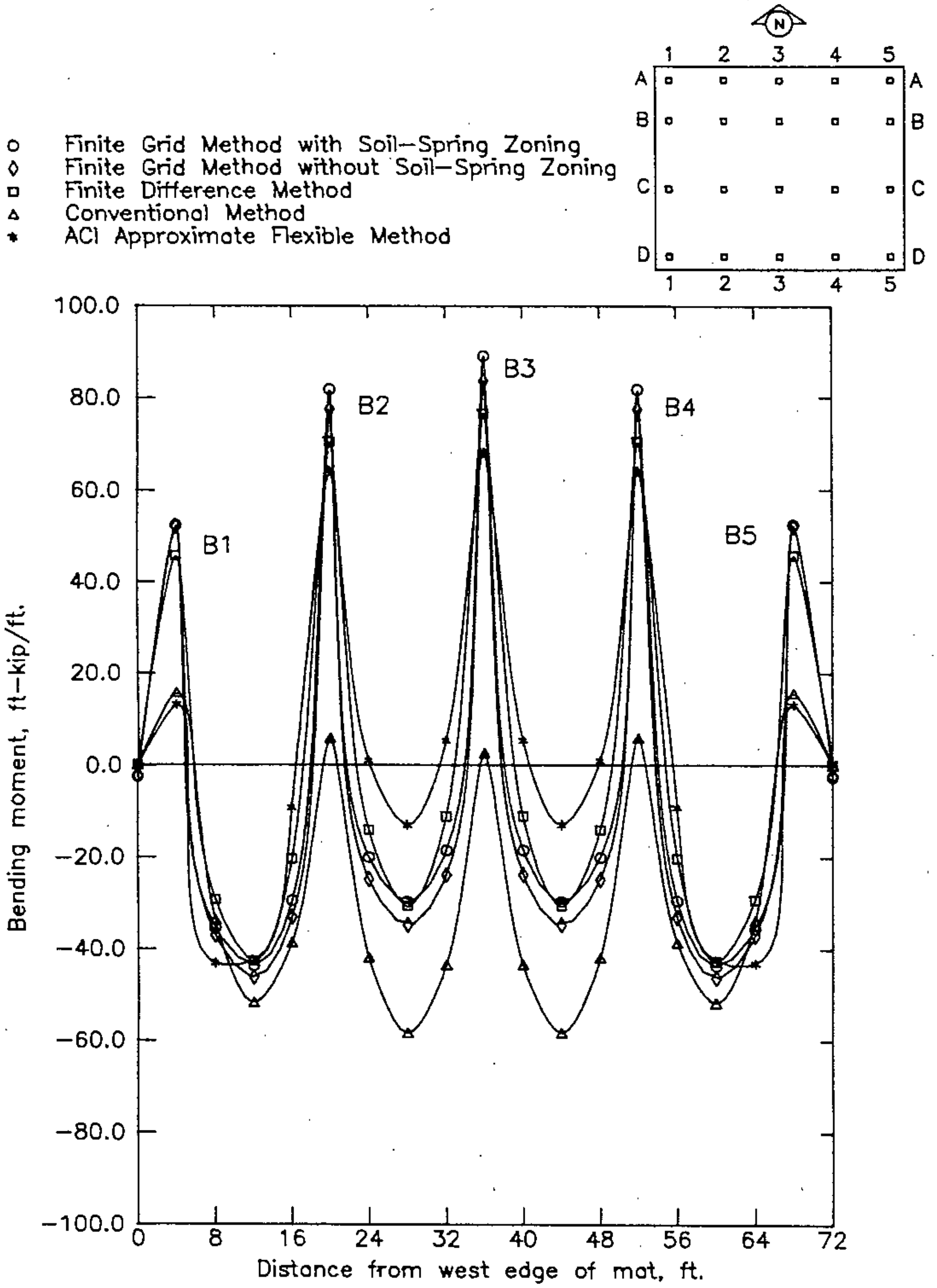

Fig.5.6 Comparison of moments along column line B

B3, the maximum positive moment is obtained by finite grid method with soil spring zoning and its value is 89.1 ft-kip/ft. Moments obtained by finite grid method without soil spring zoning, finite difference method and ACI approximate flexible method are 83.64, 76.55, 68.09 ft-kip/ft respectively. These values are less than the finite grid method with soil spring zoning by 6.1%, 14.1% and 23.6% respectively. The moment obtained by conventional method is 2.63 ft-kip/ft which is still very small compare to other methods due to reasons discussed earlier.

At column B2, the maximum moment is still given by finite grid method with soil spring zoning and it is 81.81 ft-kip/ft. Finite grid method without soil spring zoning, finite difference method and ACI flexible method yield moments having magnitude 77.43, 70.43 and 64.11 ft-kip/ft. These values are less than finite grid method with soil spring zoning by 5.4%, 13.9% and 21.6% respectively.

For column B1, the maximum moment is found for finite grid method with soil spring zoning and it is 52.44 ft-kip/ft. The moments obtained by finite grid method without zoning, finite difference method and ACI method is less than this by 0.74%, 12.8% and 75.1%. ACI flexible method yields very small values due reasons which have been discussed earlier. At this point, moment obtained by conventional method is 15.66 ft-kip/ft.

As in column line A, the negative moments between columns are highest for conventional method and after that, finite grid method without soil spring zoning gives higher values than other comparable methods. At a mid point between column B2 and B3, the negative moment obtained by finite grid method with soil spring zoning is -29.74 ft-kip/ft. The same moment obtained by finite grid without soil spring zoning and finite difference method are -34.83 and -30.61 ft-kip/ft and they are higher than the moment obtained by finite grid method with soil spring zoning by 17.1% and 2.9%, respectively. ACI flexible method yields a moment of -13.0 ft-kip/ft which is 56.3% smaller than finite grid method with soil spring zoning. Conventional method produces a moment of -58.4 ft-kip/ft.

At the mid point between column B1 and B2, the moments derived using different methods are close to one another in contrast to the interior negative moments. Here moments obtained by finite grid method with soil spring zoning is -43.72 ft-kip/ft. Finite grid method without soil spring zoning, finite difference method, ACI approximate method and conventional method yield moment of -46.35, -42.77, -42.58 and -51.88 ft-kip/ft and they are 106.0%, 97.8 %, 97.4%, 118.7% of the moment obtained by finite grid method with soil spring zoning. Moments obtained by different methods at points along column line B with their percentage variation is presented in Table 5.2.

Point	Bending moments, ft-kip/ft			
	Finite grid method with soil spring zoning	Finite grid method with out soil spring zoning	Finite difference method	ACI approximate flexible method
B1	52.44 (100.0 %)	52.05 (99.3 %)	45.72 (87.2 %)	13.04 (24.9 %)
B2	81.81 (100.0 %)	77.43 (94.6 %)	70.43 (86.1 %)	64.11 (78.4 %)
B3	89.10 (100.0 %)	83.64 (93.9 %)	76.55 (85.9 %)	68.09 (76.4 %)
B1-B2	-43.72 (100.0 %)	-46.35 (106.0 %)	-42.77 (97.8 %)	-42.58 (97.4 %)
B2-B3	-29.74 (100.0 %)	-34.83 (117.1 %)	-30.61 (102.9 %)	-12.99 (43.7 %)

Table 5.2 : Comparison of moments along column line B

The moment diagram along column line C is shown in Fig.5.7. The highest moment at column C3 is due to finite grid method with soil spring zoning and the value is 107.51 ft-kip/ft. Other methods i.e. finite grid method without soil spring zoning, finite difference method and ACI approximate flexible method give moments equal to 102.2, 92.78 and 82.67 ft-kip/ft. These values are 4.9%, 13.7% and 23.1% less than the result obtained by finite grid method with soil spring zoning, respectively.

At C2 column point, the highest moment obtained by finite grid method with soil zoning is 100.88 ft-kip/ft. Moments derived from finite grid method without soil zoning, finite difference method, ACI approximate flexible method are 96.5, 87.39 and 77.26 ft-kip/ft and they are less than finite grid with soil spring zoning by 4.3%, 13.4% and 23.4%. Moment obtained by conventional

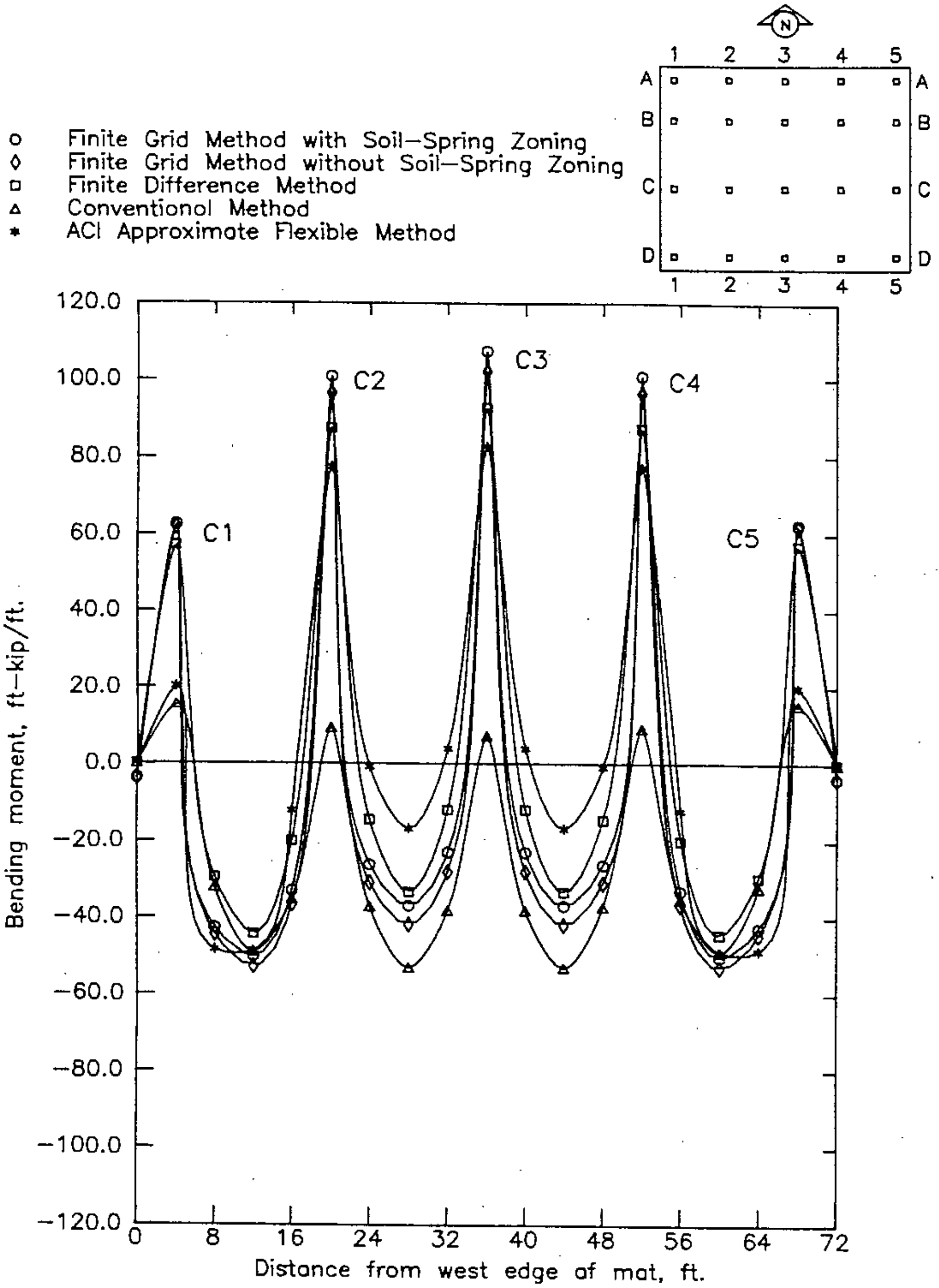

Fig.5.7 Comparison of moments along column line C

method is 9.4 ft-kip/ft which is 90.7% smaller than 100.88 ft-kip/ft.

At the exterior column C1, highest moment is 62.55 ft-kip/ft and it is still due to finite grid method with zoning. Finite grid method without zoning, finite difference method and ACI method give 61.82, 57.14, 20.05 ft-kip/ft, respectively. These are smaller than 62.55 ft-kip/ft by 1.2%, 8.6% and 67.9%. As in earlier column lines, the moment by ACI flexible method at this exterior column point is very small compared to other methods due to large effect of end conditioning forces in the vicinity of the edge of mat. The moment obtained by conventional method is 15.37 ft-kip/ft which is also very small.

For points between columns, the highest negative moment is still due to conventional method and the next higher is for finite grid without soil spring zoning. At mid point between C2-C3 and C1-C2, ACI method gives 54.6% and 2.6% less moment than moment obtained by finite grid method with soil spring zoning. The moments obtained by different methods along column line C with percentage variation among them are presented in Table 5.3

Point	Bending moments, ft-kip/ft			
	Finite grid method with soil spring zoning	Finite grid method with out soil spring zoning	Finite difference method	ACI approximate flexible method
C1	62.55 (100.0 %)	61.82 (98.8 %)	57.14 (91.4 %)	20.05 (32.1 %)
C2	100.88 (100.0 %)	96.50 (95.6 %)	87.39 (86.6 %)	77.26 (76.6 %)
C3	107.51 (100.0 %)	102.2 (95.1 %)	92.78 (86.3 %)	82.67 (76.9 %)
C1-C2	-50.32 (100.0 %)	-53.10 (105.5 %)	-44.47 (88.4 %)	-48.99 (97.4 %)
C2-C3	-37.04 (100.0 %)	-42.00 (113.4 %)	-33.45 (90.3 %)	-16.82 (45.4 %)

Table 5.3 : Comparison of moments along column line C

The moment diagram along column line D is presented in Fig.5.8 . At D3, the highest moment is due to finite grid method with soil spring zoning and it is

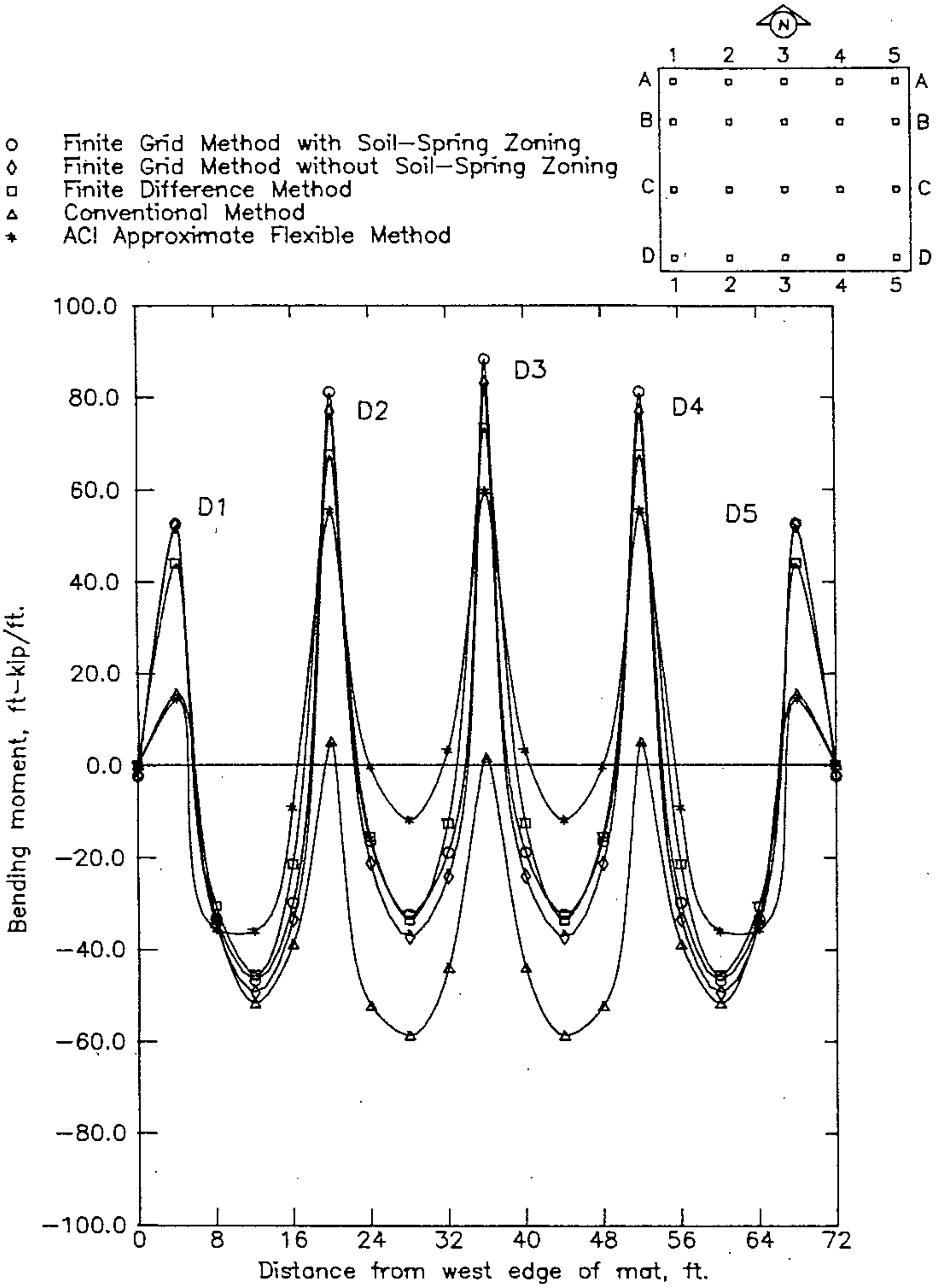

Fig.5.8 Comparison of moments along column line D

88.3 ft-kip/ft. Finite grid method without zoning, finite difference method and ACI approximate flexible method give results less than this by 6.1%, 16.9% and 32.5% respectively. At column point D2 , finite grid method with soil zoning yields a moment of 81.10 ft-kip/ft and finite grid without zoning, finite difference, ACI flexible methods produce moment which are 5.3%, 16.8% and 31.7% less than 81.10 ft-kip/ft. At column D1, the moment obtained by finite grid method without zoning, finite difference and ACI method are 0.7%, 16.2%, 72.7% less than moment obtained by finite grid method with soil spring zoning which is 52.58 ft-kip/ft.

For the mid point between column D1 and D2, the negative moment obtained by finite grid method with soil spring zoning is -46.87 ft-kip/ft. Finite grid method without zoning and conventional method yield moments 5.7% and 10.6% higher than -46.87 ft-kip/ft. Finite difference and ACI methods produce moment 2.5% and 22.9% lesser in magnitude than the same. For the middle point between column D2 and D3, moment obtained by finite grid method with zoning is -32.4 ft-kip/ft. Moments obtained by finite grid method without zoning, finite difference and conventional methods are 15.7%, 4.1%, 81.4% higher and that obtained by ACI approximate flexible method is 63.3% lesser in magnitude than this 32.4 ft-kip/ft. Moments obtained by different methods along column line D with their percentage variation are presented in Table 5.4.

Point	Bending moments, ft-kip/ft			
	Finite grid method with soil spring zoning	Finite grid method with out soil spring zoning	Finite difference method	ACI approximate flexible method
D1	52.58 (100.0 %)	52.20 (99.3 %)	44.04 (83.8 %)	14.33 (27.3 %)
D2	81.10 (100.0 %)	76.80 (94.7 %)	67.50 (83.2 %)	55.40 (68.3 %)
D3	88.30 (100.0 %)	82.98 (93.9 %)	73.35 (83.1 %)	59.59 (67.5 %)
D1-D2	-46.87 (100.0 %)	-49.53 (105.7 %)	-45.68 (97.5 %)	-36.15 (77.1 %)
D2-D3	-32.40 (100.0 %)	-37.50 (115.7 %)	-33.72 (104.1 %)	-11.89 (36.7 %)

Table 5.4 : Comparison of moments along column line D

Moments along column lines 1, 2, 3 are shown in Fig.D.1, D.2 and D.3 of appendix D respectively and they are compared in Table 5.5, 5.6 and 5.7 with percentage variation among different methods.

Point	Bending moment, ft-kip/ft			
	Finite grid method with soil spring zoning	Finite grid method with out soil spring zoning	Finite difference method	ACI approximate flexible method
A1	39.94 (100.0 %)	39.33 (98.5 %)	34.94 (87.5 %)	10.46 (26.5 %)
B1	75.77 (100.0 %)	71.30 (94.1 %)	62.00 (81.8 %)	46.58 (61.5 %)
C1	101.14 (100.0 %)	95.43 (94.4 %)	86.73 (85.8 %)	72.66 (71.8 %)
D1	51.42 (100.0 %)	50.9 (98.9 %)	42.99 (83.6 %)	17.17 (33.4 %)

Table 5.5 : Comparison of moments along column line 1

Point	Bending moment, ft-kip/ft			
	Finite grid method with soil spring zoning	Finite grid method with out soil spring zoning	Finite difference method	ACI approximate flexible method
A2	45.19 (100.0 %)	44.40 (98.3 %)	40.28 (89.1 %)	13.75 (30.4 %)
B2	84.32 (100.0 %)	79.60 (94.4 %)	71.96 (85.3 %)	60.85 (72.2 %)
C2	114.73 (100.0 %)	109.20 (95.2 %)	100.04 (87.2 %)	96.14 (83.8 %)
D2	57.06 (100.0 %)	56.40 (98.8 %)	51.42 (90.1 %)	21.54 (37.7 %)

Table 5.6 : Comparison of moments along column line 2

Point	Bending moment, ft-kip/ft			
	Finite grid method with soil spring zoning	Finite grid method with out soil spring zoning	Finite difference method	ACI approximate flexible method
A3	44.99 (100.0 %)	44.05 (97.9 %)	40.72 (90.5 %)	14.22 (31.6 %)
B3	84.41 (100.0 %)	79.50 (94.2 %)	73.04 (86.5 %)	61.37 (72.7 %)
C3	115.02 (100.0 %)	109.60 (95.3 %)	101.46 (88.2 %)	97.10 (84.4 %)
D3	57.08 (100.0 %)	56.30 (98.6 %)	51.89 (90.9 %)	21.9 (38.4 %)

Table 5.7 : Comparison of moments along column line 3

It is found from all the moment diagrams that finite grid method with soil spring zoning yields higher positive moments at column points than other methods. But at mid point between columns, the negative moments obtained by both finite grid method without zoning and finite difference method are higher than that obtained by finite grid method with soil spring zoning. This implies that soil spring zoning accumulates positive moments at column points by reducing negative moments at points between columns. Soil spring zoning introduces softer soil springs towards the centre of the mat and the stiffness contributed by the springs are, thereby, reduced. But results show that the stiffness reduction of springs are more pronounced at points between columns than at columns. The reduction of stiffness at column points being lesser, they take higher share in comparison to middle points.

This variation between finite grid method with and without zoning can be explained using moment curvature relations. In the deflection curves of mat (Fig.5.9, 5.10 and Fig.D.4 to D.10 of appendix D), it is clear that the slope of the curve, which is the curvature, becomes flatter for finite grid method with zoning than the same without zoning at points between the columns. On the other hand, at column points, this curvature is higher for finite grid method with zoning. As moment is directly proportional to curvature, it is not surprising that finite grid method with soil spring zoning yields higher moments at column points and lesser at points between columns compared to the same method using no soil spring zoning.

It is also evident that the effect of zoning is more pronounced at interior columns than exteriors and it is due to the fact that zoning makes softer spring towards the centre from all directions.

Finite difference method yields moments at columns about 8 to 18% less than finite grid method with soil zoning; higher variation being found towards the centre due to softer soil springs. As for negative moments between column points, finite difference produces 48% less to 17% higher moments than finite grid with soil spring zoning. If finite difference method is compared to finite grid method without zoning, it creates positive moments at column points 7 to 17% less than the moments found by the latter method. Finite difference method yields less negative moments between column points than moments obtained by finite grid method without soil spring zoning. ACI approximate flexible method gives better results at column points away from edges and at these points moments are about 15 to 30% less than the same obtained by finite grid method with soil spring zoning. The poor performance of ACI method near edges are due to the concentrated effect of the end conditioning forces.

5.3.2 Deflections along Column Lines

Deflections obtained by different methods along column line A and C are presented in Fig.5.9 and column line 1 and 3 in Fig.5.10. Also deflections of all the column lines are separately presented in Fig.D.4 to D.10 of appendix D. Deflections at column points and midpoint between column are presented in Tables 5.8 to 5.11. In these tables, the deflections obtained by different methods are shown with their percentage variation with finite grid method with soil spring zoning within bracket .

The mat is symmetrical in east-west directions and deflection curve along column line A, B, C, D are also symmetrical. Deflections along column line 4, 5 are not presented because they are identical to column line 2, 1, respectively, due to symmetry. Deflection curves show that finite grid method without zoning and finite difference method give almost equal deflection at all the points; both these methods consider uniform soil springs under the mat. The results of finite grid method with soil spring zoning clearly indicate the effect of zoning. In this method, the deflections of interior points are much higher than the deflections obtained from of finite grid method without zoning and finite difference method. At column line A, D, 1, the deflections at the edges

Point	Deflection, inch			
	Finite grid method with soil spring zoning	Finite grid method with out soil spring zoning	Finite difference method	ACI approximate flexible method
A1	0.28464 (100.0 %)	0.30240 (106.7 %)	0.30376 (106.7 %)	0.31853 (111.9 %)
A2	0.28572 (100.0 %)	0.26676 (93.4 %)	0.26934 (94.2 %)	0.15064 (52.7 %)
A3	0.29976 (100.0 %)	0.26676 (89.0 %)	0.26956 (89.9 %)	0.18568 (61.9 %)
A1-A2	0.26844 (100.0 %)	0.26592 (99.1 %)	0.26425 (98.4 %)	0.16235 (60.5 %)
A2-A3	0.28620 (100.0 %)	0. 25680 (89.7 %)	0.25585 (89.4 %)	0.16384 (57.2 %)

Table 5.8 : Comparison of deflections along column line A

Point	Deflection, inch			
	Finite grid method with soil spring zoning	Finite grid method with out soil spring zoning	Finite difference method	ACI approximate flexible method
B1	0.27276 (100.0 %)	0.26652 (97.7 %)	0.27005 (99.0 %)	0.42717 (156.6 %)
B2	0.27732 (100.0 %)	0.23532 (84.9 %)	0.24044 (86.7 %)	0.20170 (72.7 %)
B3	0.29172 (100.0 %)	0.23592 (80.9 %)	0.24088 (82.6 %)	0.24848 (85.2 %)
B1-B2	0.25656 (100.0 %)	0.23052 (89.9 %)	0.23133 (90.2 %)	0.21690 (84.5 %)
B2-B3	0.27564 (100.0 %)	0. 22332 (81.0 %)	0.22439 (81.4 %)	0.21728 (78.8 %)

Table 5.9 : Comparison of deflections along column line B

Point	Deflection, inch			
	Finite grid method with soil spring zoning	Finite grid method with out soil spring zoning	Finite difference method	ACI approximate flexible method
C1	0.25692 (100.0 %)	0.24180 (94.1 %)	0.24639 (95.9 %)	0.45028 (175.3 %)
C2	0.26436 (100.0 %)	0.21492 (81.3 %)	0.22194 (84.0 %)	0.20253 (76.6 %)
C3	0.27888 (100.0 %)	0.21612 (77.5 %)	0.22294 (79.9 %)	0.25209 (90.4 %)
C1-C2	0.24012 (100.0 %)	0.20604 (85.8 %)	0.20846 (86.8 %)	0.21949 (91.4 %)
C2-C3	0.26028 (100.0 %)	0.20088 (77.2 %)	0.20361 (78.2 %)	0.21660 (83.2 %)

Table 5.10 : Comparison of deflections along column line C

Point	Deflection, inch			
	Finite grid method with soil spring zoning	Finite grid method with out soil spring zoning	Finite difference method	ACI approximate flexible method
D1	0.28728 (100.0 %)	0.30588 (106.5 %)	0.30693 (106.8 %)	0.32857 (114.4 %)
D2	0.29088 (100.0 %)	0.27384 (94.1 %)	0.27814 (95.6 %)	0.14142 (48.6 %)
D3	0.30516 (100.0 %)	0.27444 (89.9 %)	0.27871 (91.3 %)	0.17752 (57.5 %)
D1-D2	0.27000 (100.0 %)	0.26880 (99.6 %)	0.26847 (99.4 %)	0.15552 (57.6 %)
D2-D3	0.28872 (100.0 %)	0.26160 (90.6 %)	0.26200 (90.7 %)	0.15160 (52.5 %)

Table 5.11 : Comparison of deflections along column line D

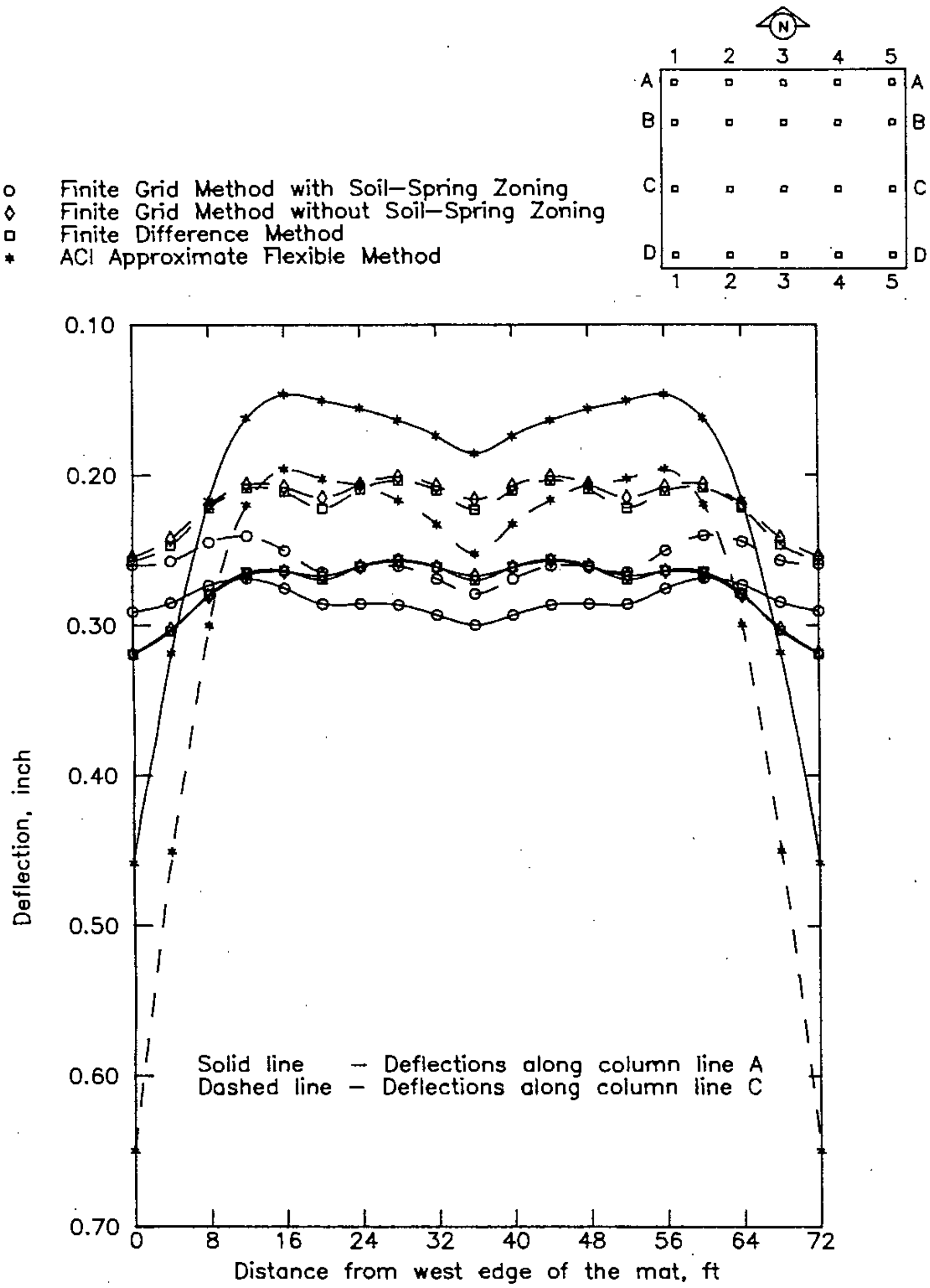

Fig.5.9 Comparison of deflections along column line A and C

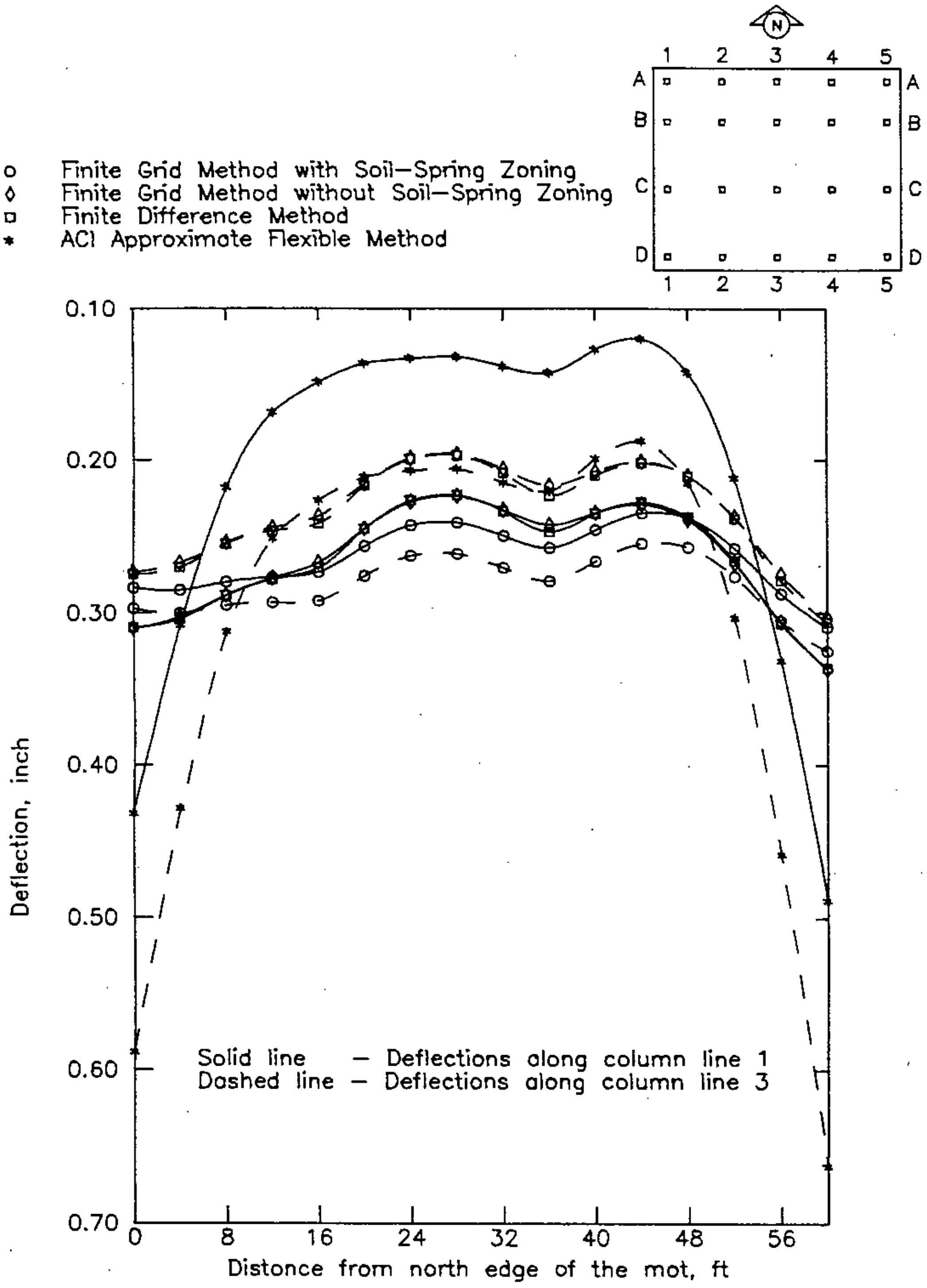

Fig.5.10 Comparison of deflections along column line 1 and 3

are lower for finite grid method with zoning in comparison to its without zoning counterpart. This may be visualised as the uplift of a corner of plate which is deflected in a dished shape. At column line A, B, C and D, the edge deflections of mat from finite grid without zoning and finite difference method are higher than the centre deflections. In shorter directions i.e. along 1, 2 and 3, all the methods display edge deflections higher than central deflections. The deflected shape of mat can be visualised as the top of a pillow.

ACI flexible method yields very high deflections at edges along all the column lines due to the concentrated effect of end conditioning forces. At exterior column lines i.e. A, D and I, the deflection of interior columns are quite small compared to other methods. When deflections at line A is calculated, the mat is first assumed to be infinite in all directions and to correct this assumption, end conditioning forces are applied at east and west edges considering a beam of unit width. But the mat still remains infinite on north edge, as a result the deflections of inner columns along column line A are smaller and the same is true for column line D, 1 and 5. For interior column lines, the deflections of points away from edge are more or less comparable to finite grid and finite difference methods.

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5.4 PARAMETRIC STUDY

5.4.1 Effect of Modulus of Subgrade Reaction

Effect on moment

To find the effect of modulus of subgrade reaction of soil on the behaviour of mat, the modulus is varied from 50 to 300 kcf keeping all other geometric and material properties unchanged. Four column points (A1, A3, C1, C3) on the mat are selected where moments in both directions are to be compared. It may be mentioned here that modulus of subgrade reaction is used to calculate soil spring values and higher modulus means stiffer soil. It can be easily visualised that as the supporting soil become stiffer, the load transferred from column to soil will be more concentrated around the column and moments at column points will be higher. But it should also be noted that the mat rigidity is very high compared to the soil stiffness and as a result, the effect of soil stiffness should not be significant.

The moments M_x (as shown in Fig.2.1) at column point A1 obtained by finite grid method with and without soil spring zoning, finite difference method and ACI approximate flexible method for different modulus of subgrade reaction of soil are presented in Fig.5.11. These methods give 2.5%, 3.5%, 3.9% and 92.9% higher values for an increase in the magnitude of soil modulus from 50 kcf to 300 kcf. The results of ACI method is not at all comparable due to the concentrated effect of end conditioning forces at this corner column.

Moment in x-direction at column A3 is presented in Fig.5.12. Here moments are increased by 3.0% and 3.4% for finite difference and finite grid without soil zoning for a change in the value the modulus from 50 to 300 kcf. But for finite grid method with soil spring zoning, 8.9% lesser moment is obtained for the same change in the modulus. Although increased modulus means stiffer soil, soil zoning makes softer springs at A3 than at A1 and, as high load is transferred to stiffer portion, moment at A3 is reduced. Moment obtained by ACI method is reduced by 25.9% for 300 kcf than for 50 kcf. This result is also not comparable.

Moments in x direction at column C1 are presented in Fig.5.13. Moments obtained by finite grid method with and without zoning, finite difference method and ACI flexible method are 2.0%, 2.9%, 3.2% and 43.9% higher, respectively for 300 kcf in comparison to results for 50 kcf soil modulus. ACI method, once again, yields incomparable results.

For column point C3, moments M_x are presented in Fig.5.14. Here, finite grid without zoning and finite difference method yield 1.1% and 3.3% higher moments for 300 kcf than for 50 kcf. But finite grid method with soil zoning yields 6.1% less moment for 300 kcf as in column point A3. ACI method gives 20.3% less moment for 300 kcf soil modulus.

Moments M_y (as shown in Fig.2.1) at column A1 are presented in Fig.5.15. Here moments obtained by finite grid with and without zoning and finite difference method give 3.7%, 2.3% and 2.6% less moments for 300 kcf than for 50 kcf. This is due to more stiffening of interior columns than exterior ones and moments are transferred to stiffer portion.

At point A3, the moments M_y are shown in Fig.5.16. Here finite grid with zoning yields 1.7% less moment for 300 kcf than for 50 kcf, as in M_x at the

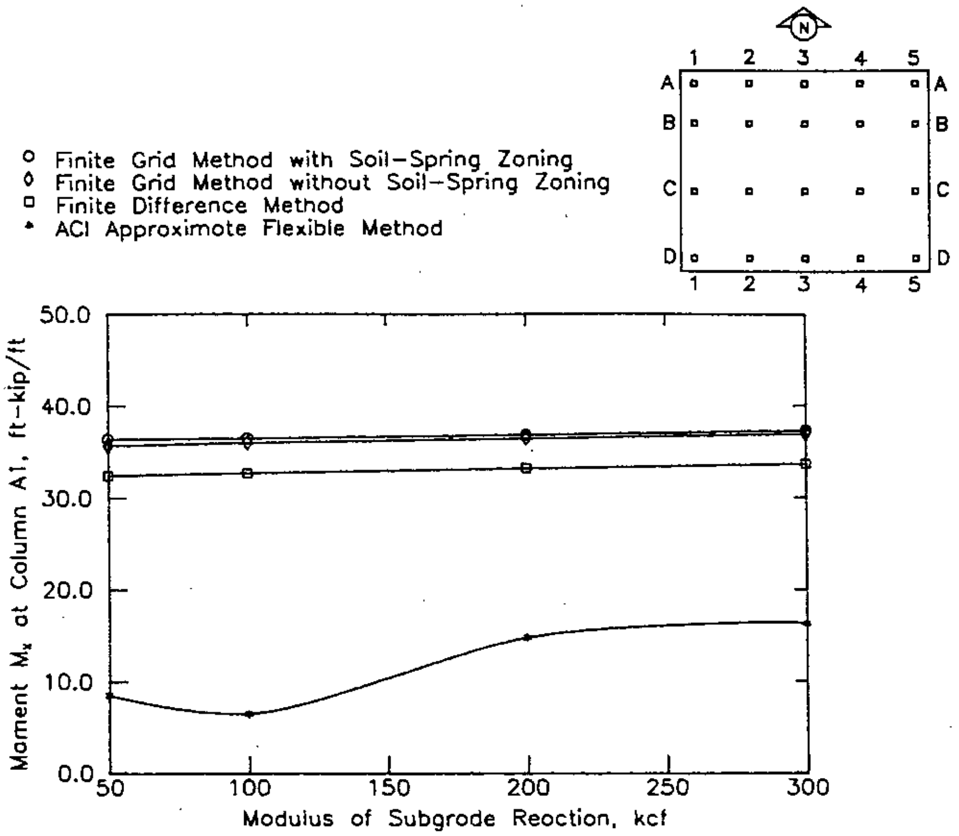


Fig.5.11 Voriotion of moment M_x ot column A1 with modulus of subgrode reoction

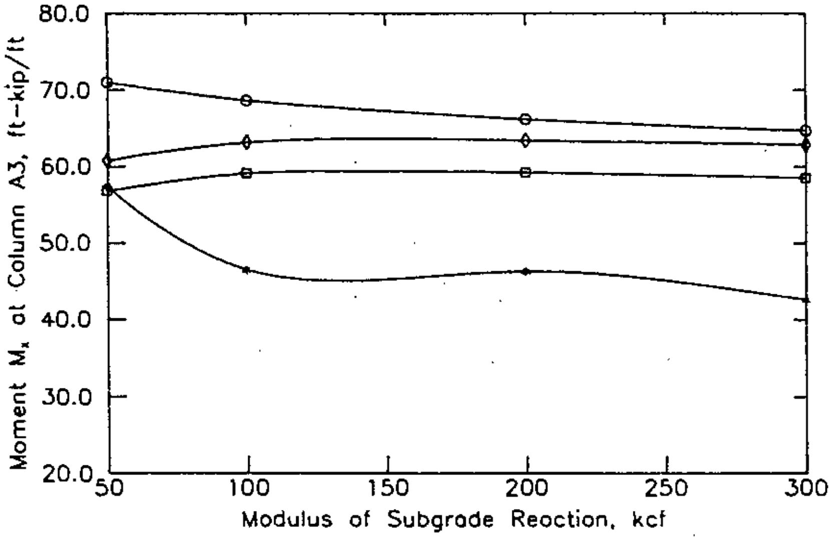


Fig.5.12 Voriotion of moment M_x ot column A3 with modulus of subgrode reoction

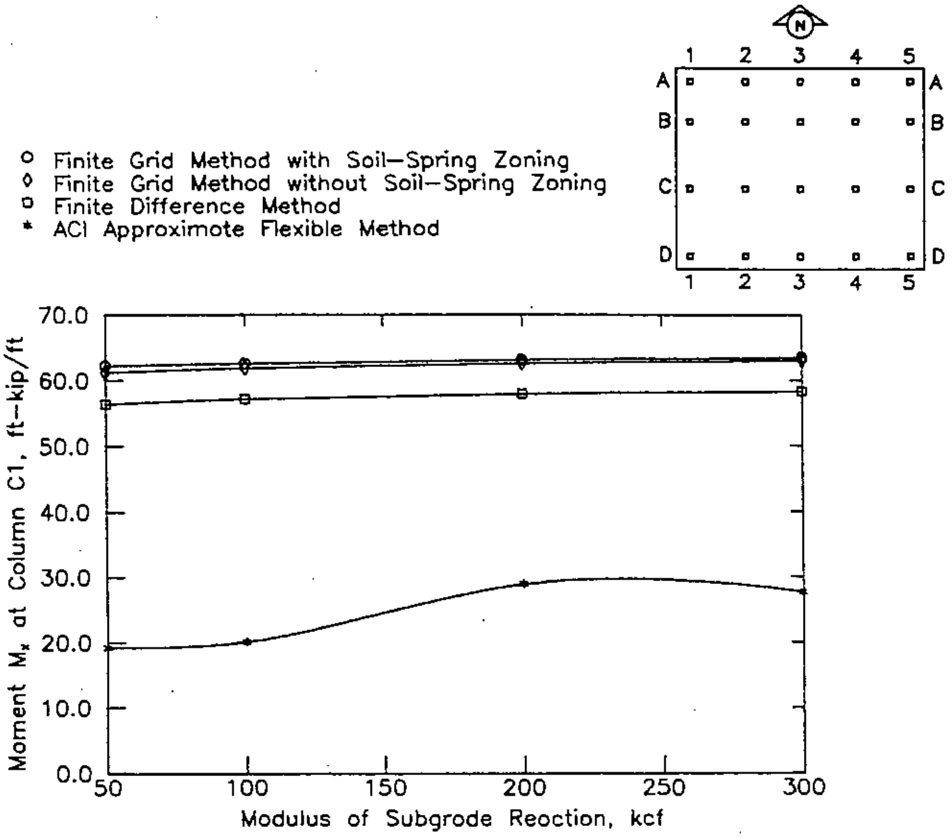


Fig.5.13 Voriotion of moment M_x ot column C1 with modulus of subgrode reoction

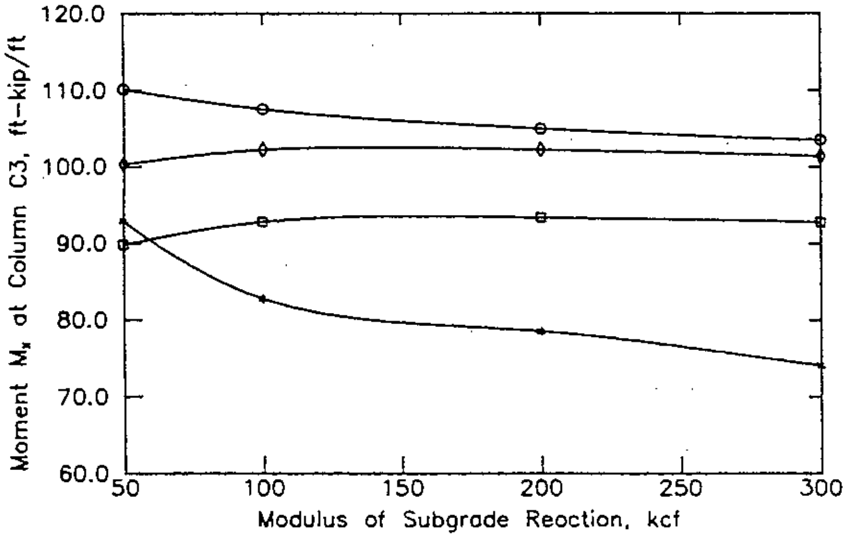


Fig.5.14 Voriotion of moment M_x ot column C3 with modulus of subgrode reaction

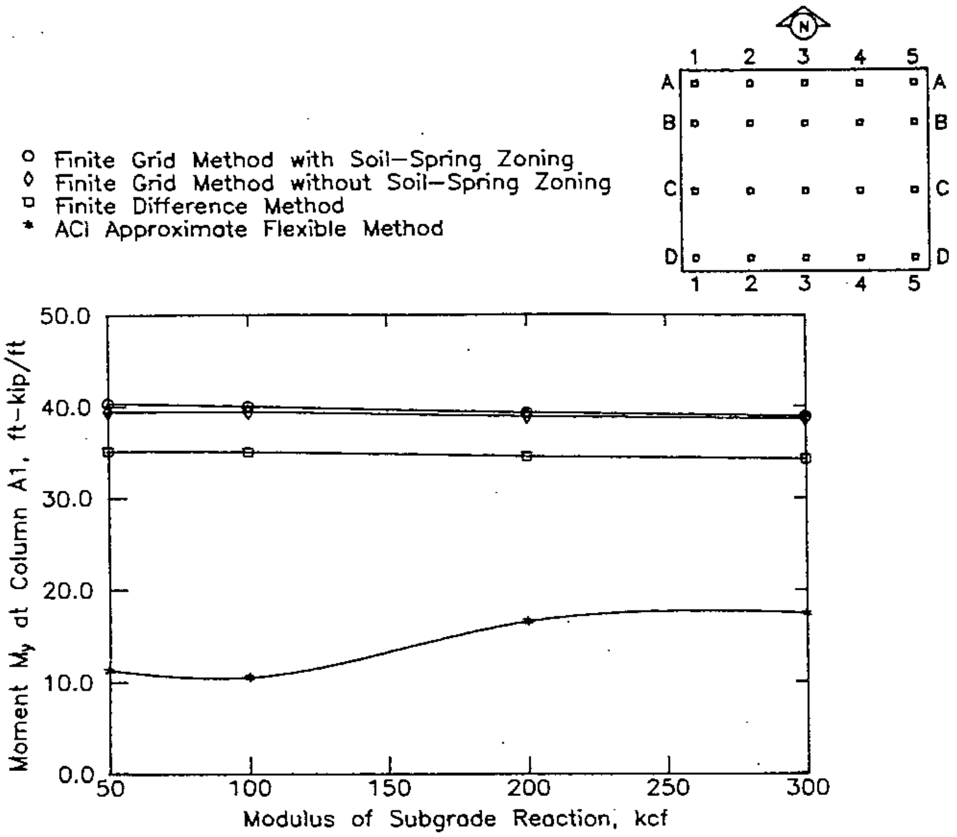


Fig.5.15 Variation of moment M_y at column A1 with modulus of subgrade reaction

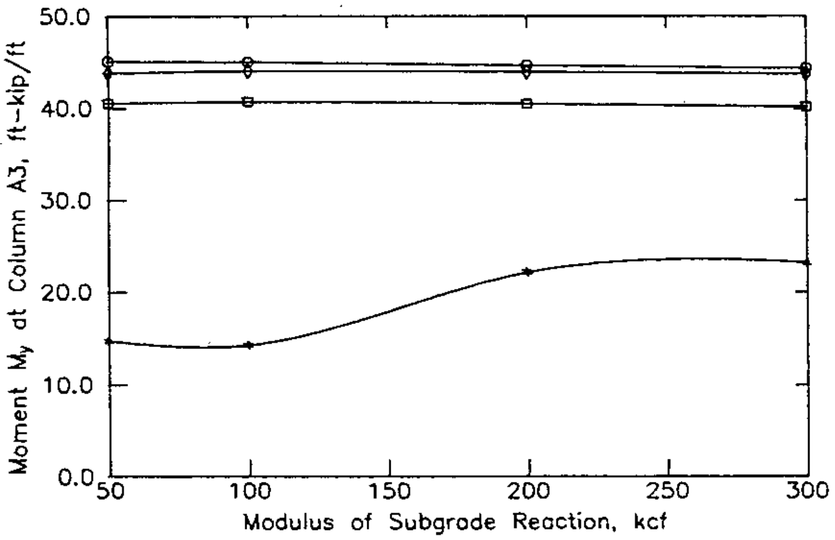


Fig.5.16 Variation of moment M_y at column A3 with modulus of subgrade reaction

same point. Both finite grid without zoning and finite difference methods yield similar variation in moment. They are slightly increased at 100 kcf and then decreased slowly for 200 and 300 kcf soil modulus.

Moments M_y at column C1 are shown in Fig.5.17. Finite grid method with and without soil spring zoning and finite difference methods yield 7.1%, 16.6% and 17.4% higher moments, respectively, due to an increase in k_s from 50 to 300 kcf. ACI flexible method results are more or less comparable but there variation with modulus is not clear since the end conditioning forces are also changing with change in k_s.

At column C3, moments M_y are presented in Fig.5.18. Finite grid method with and without zoning, finite difference and ACI method give 6.0%, 13.0% and 13.6% higher moments for 300 kcf than for 50 kcf. These variations for finite grid without zoning and finite difference are the highest of all other points. ACI method yield comparable results and but moment for 300 kcf is 6.7% smaller than the same for 50 kcf.

Effect on deflection

Deflections at column point A1 are presented in Fig.5.19. With the increase in subgrade modulus, soil gets stiffer and deflections becomes smaller. Deflections obtained by finite grid method with and without soil spring zoning, finite difference method and ACI approximate method for soil subgrade modulus 300 kcf are 82.5%, 81.8%, 81.5% and 82.8% less than the same obtained for 50 kcf, respectively.

For column point A3, deflections are shown in Fig.5.20. For a modulus of 300 kcf, finite grid method with and without zoning, finite difference method and ACI method yield deflections 82.0 %, 81.7 %, 81.3 % and 76.7 % less than the same for a modulus of 50 kcf.

Deflections at column point C1 are shown in Fig.5.21. Deflections obtained by finite grid method with and without soil zoning, finite difference method and ACI method for 300 kcf soil modulus are 80.7 %, 80.5 %, 79.6 % less than the same obtained for 50 kcf.

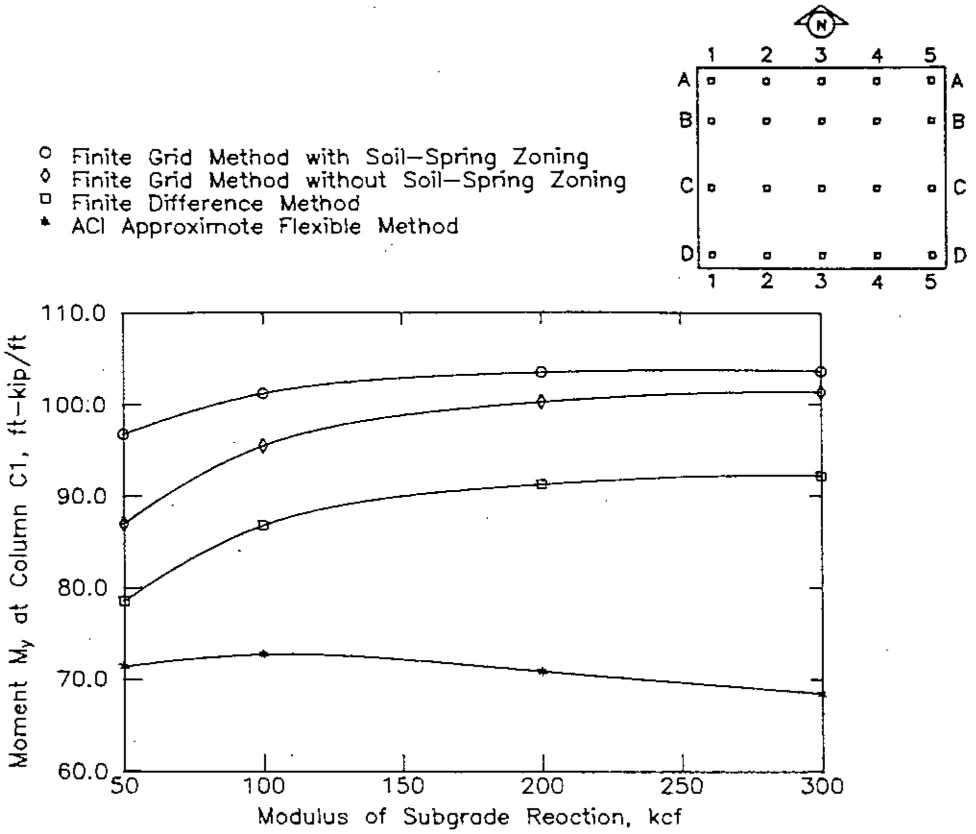


Fig.5.17 Variation of moment M_y at column C1 with modulus of subgrade reaction

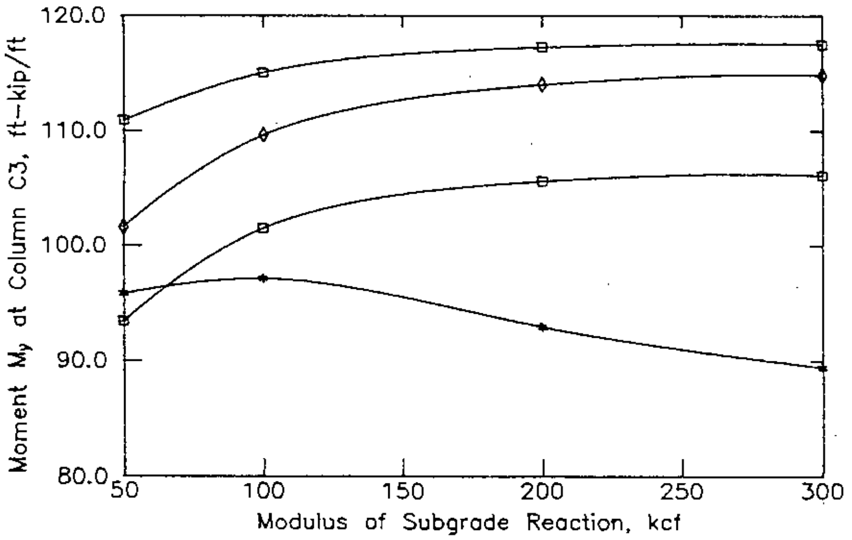


Fig.5.18 Variation of moment M_y at column C3 with modulus of subgrade reaction

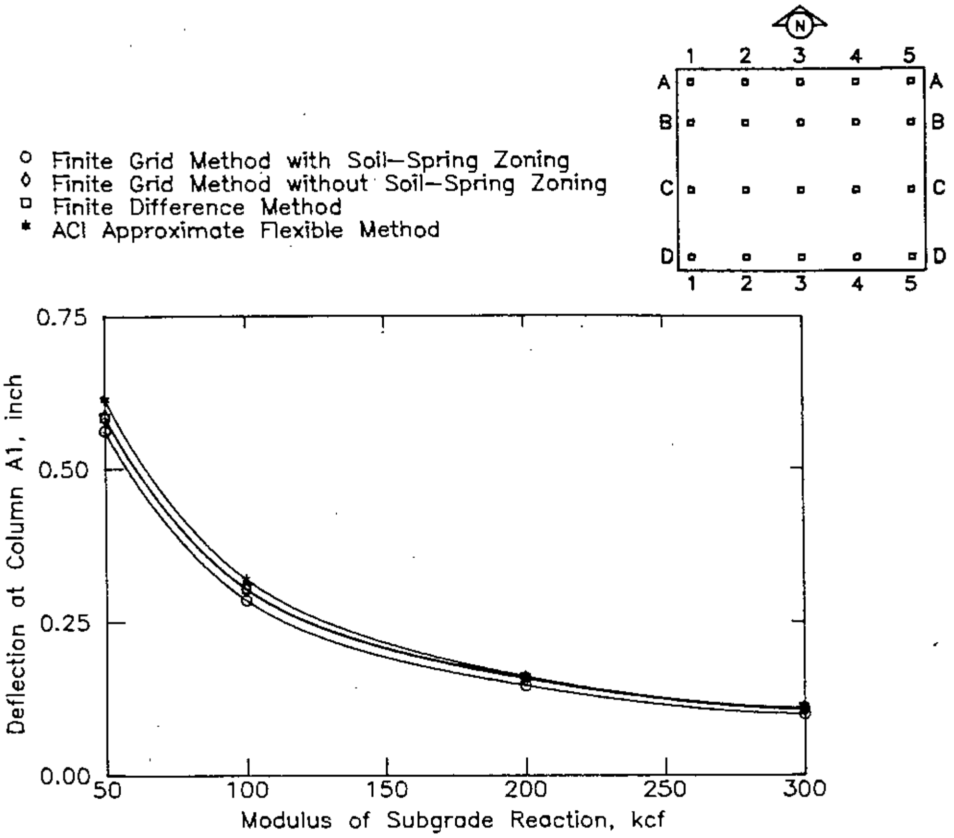


Fig.5.19 Variation of deflection at column A1 with modulus of subgrade reaction

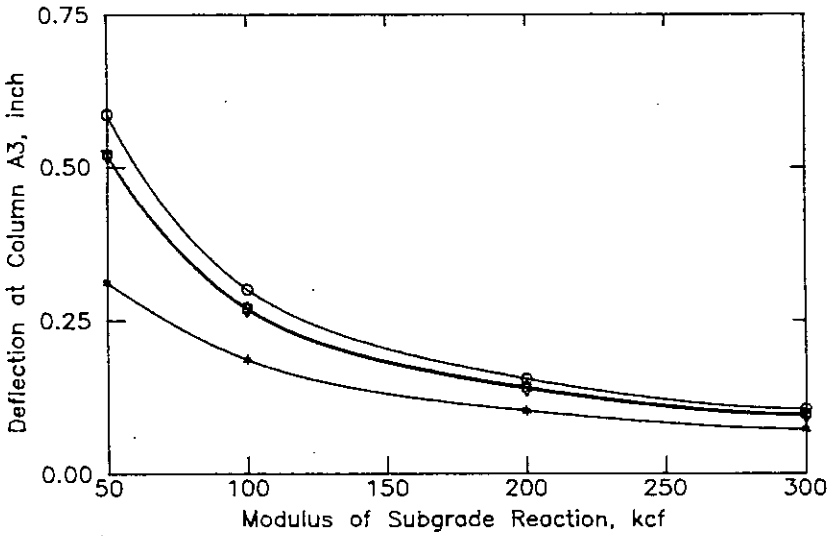


Fig.5.20 Variation of deflection at column A3 with modulus of subgrade reaction

For column point C3, deflections are presented in Fig.5.22. For a modulus of 300 kcf, finite grid method with and without soil zoning, finite difference method and ACI method yield deflections 80.0%, 79.6%, 78.6% and 77.4% less than the same for a modulus of 50 kcf.

The effect of modulus of subgrade reaction on deflection of mat is almost similar for all the methods. It has been found that although soil modulus affects very little on moments, it has a profound effect on deflection. Curves show that deflection of a point decreases almost exponentially with the increase in soil modulus.

5.4.2 Effect of Stiffness of Mat

Effect on moment (due to change in thickness)

To find the effect of mat thickness on the behaviour of mat foundation, thickness is varied from 30 to 48 inch keeping all other geometric and material properties unchanged. All these thicknesses are safe from punching point of view. Four column points (A1, A3, C1, C3) on the mat are selected where moments in both directions are to be compared. Flexural rigidity of mat increases with an increase in mat thickness and as the mat becomes stiffer, the load transferred from column to soil become more evenly distributed around the column. As a result, moments at column points become smaller.

The moments M_x at column point A1 are shown in Fig.5.23. Moments obtained by finite difference method decreases by 1.3% due to a change in mat thickness from 30 inch to 48 inch. Finite grid method with and without zoning do not show any substantial change. ACI method results are not comparable due to the effect of end conditioning forces, as mentioned earlier.

Moments M_x at column point A3 are presented in Fig.5.24. Finite difference and finite grid without soil spring zoning gives moments 12.1% and 11.2% lesser for 48 inch mat than for 30 inch mat. But finite grid method with soil spring zoning yields moments 8.0% higher for 48 inch than the same for 30 inch. Here although thickness increase results in better distribution of load in mat, soil spring zoning makes column points even stiffer resulting in higher moments at column points. ACI method also yields moments which are higher for increased thickness.

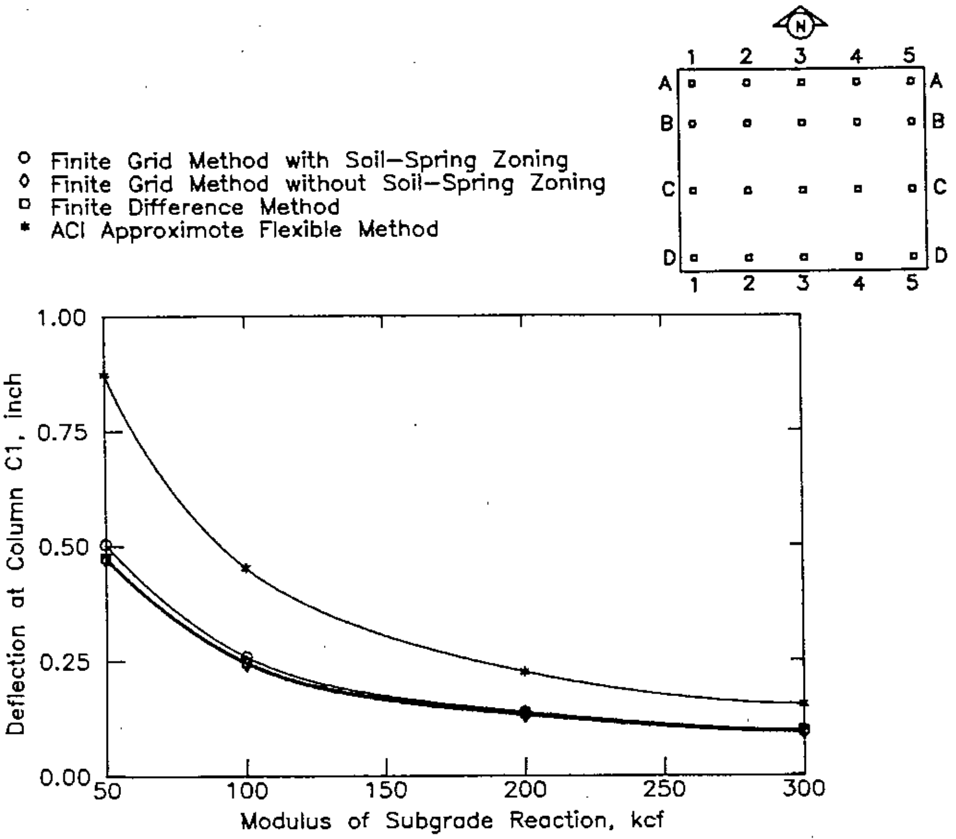


Fig.5.21 Voriation of deflection ot column C1 with modulus of subgrode reaction

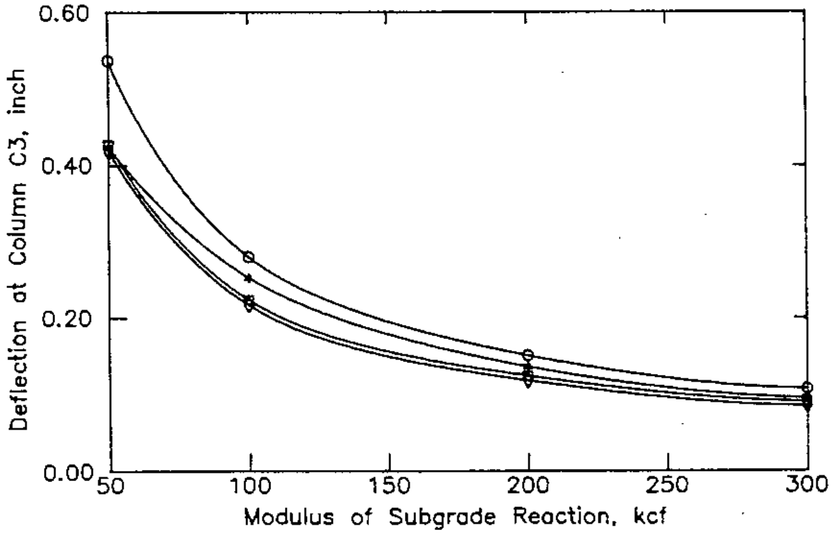


Fig.5.22 Voriotion of deflection at column C3 with modulus of subgrade reoction

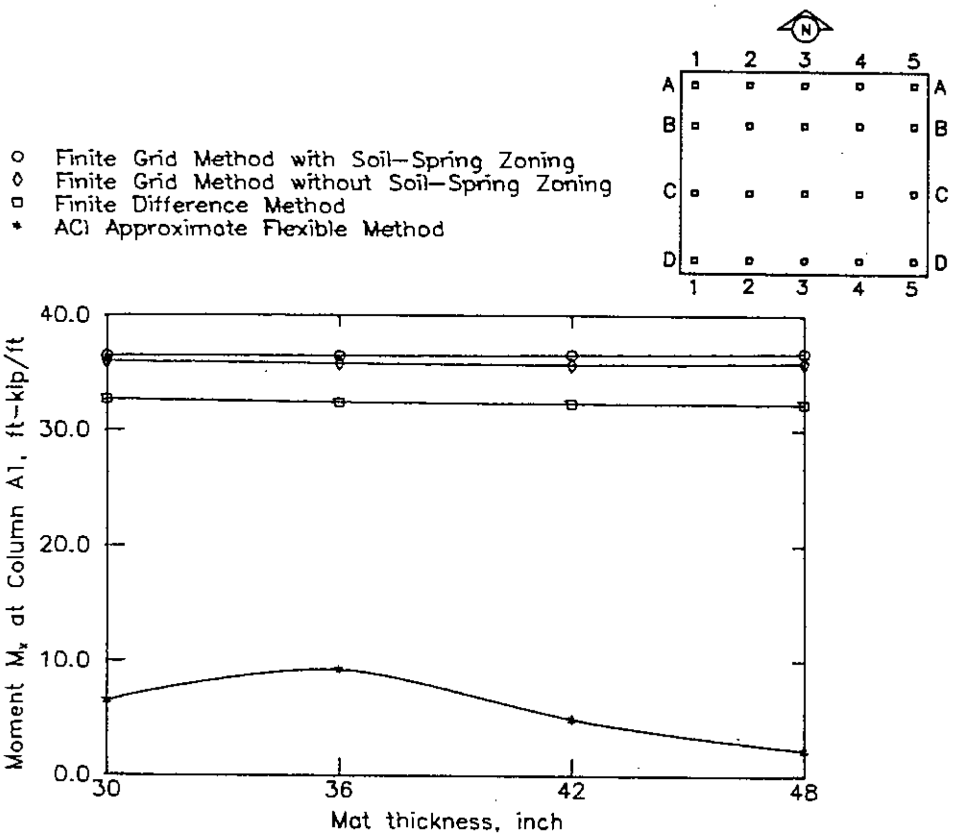


Fig.5.23 Variation of moment M_x at column A1 with mat thickness

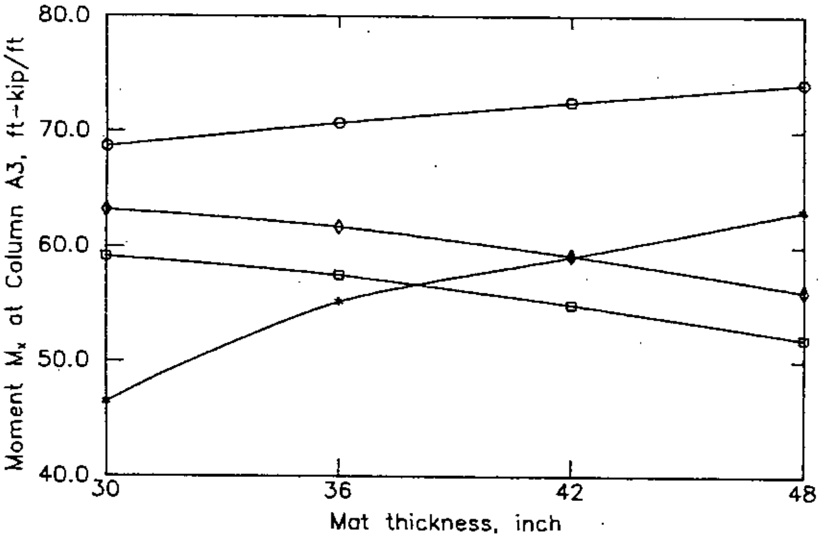


Fig.5.24 Variation of moment M_x at column A3 with mat thickness

For column point C1, moments M_x are presented in Fig.5.25. Finite difference results are lesser for 48 inch than for 30 inch by 2.8%. While, finite grid method without zoning yields very little or no variation in moment, but finite grid method with zoning results in 2.0% higher moments for 48 inch thickness than for 30 inch thick mat.

Moments M_x at column C3 are shown in Fig.5.26. Finite difference and finite grid without zoning results are lower for 48 inch thick mat by 9.6% and 3.9% and that for finite grid with zoning is 7.5% higher when compared to 30 inch thick mat.

Moments M_y at column A1 and A3 are presented in Fig.5.27 and 5.28 they give similar results as above.

Moments M_y at column C1 and C3 are presented in Fig.5.29 and 5.30. At these points, all the methods yield lesser moments for higher thickness and the decrease is substantial. At column C1, moments obtained by finite grid with and without zoning, finite difference and ACI methods are 8.9%, 19.7%, 22.4% and 9.9% lesser for the mat with 48 inch thickness compared to results for 30 inch thick mat. At column C3, reduction in moment are 6.8%, 15.8%, 19.2% and 7.3%, respectively, for the above mentioned methods due to the increase of mat thickness.

Effect on deflection (due to change in thickness)

Deflections at all the column points are decreased for higher thickness. Deflections at column point A1, A3, C1, C3 are presented in Fig.5.31 to 5.34. At A1, finite grid with and without zoning, finite difference and ACI method yield deflections 2.8%, 7.3%, 8.1% and 3.9% less for 48 inch thick mat than the same for 30 inch thick mat. At A3, the methods produce 4.6%, 5.5%, 6.2% and 30.9% less deflections for 48 inch mat. At C1, the reductions are 2.3%, 3.4%, 4.9% and 2.1%, respectively, and for C3, the reductions are 5.3%, 2.7%, 4.3% and 35.7% for the methods and thickness under consideration.

Effect on moment (due to variation in f'_c)

Concrete strength is used to calculate the modulus of elasticity of concrete. To find the effect of concrete strength (f'_c) on moment, f'_c is varied from 2500 to

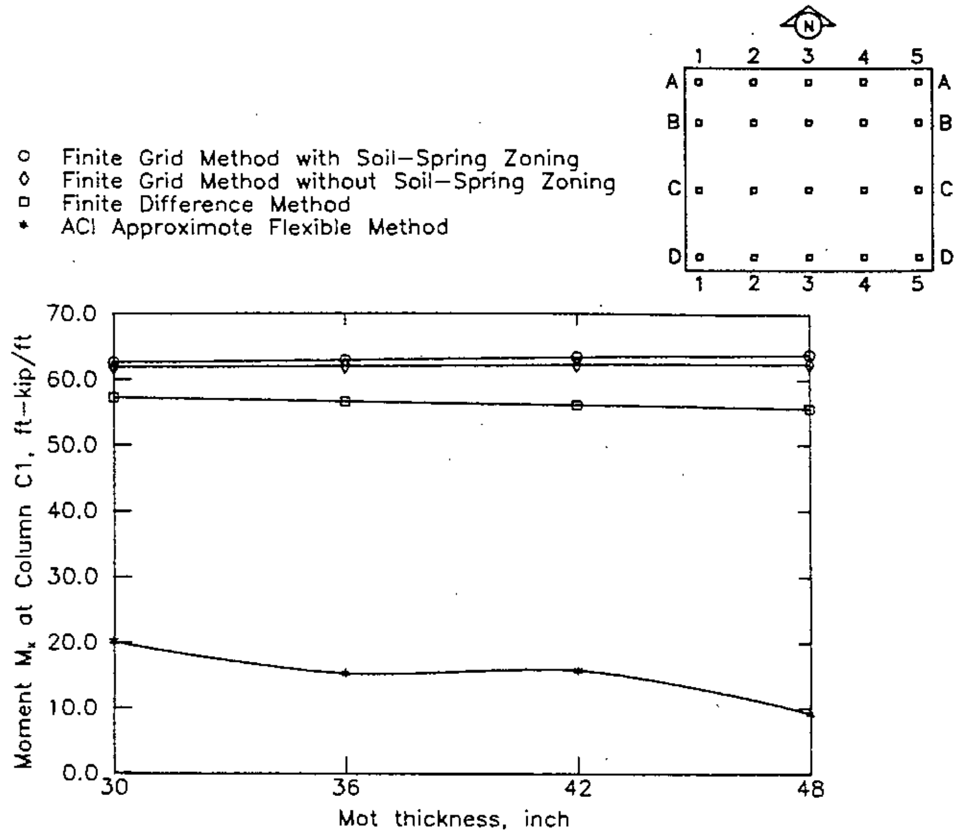


Fig.5.25 Variation of moment M_x at column C1 with mat thickness

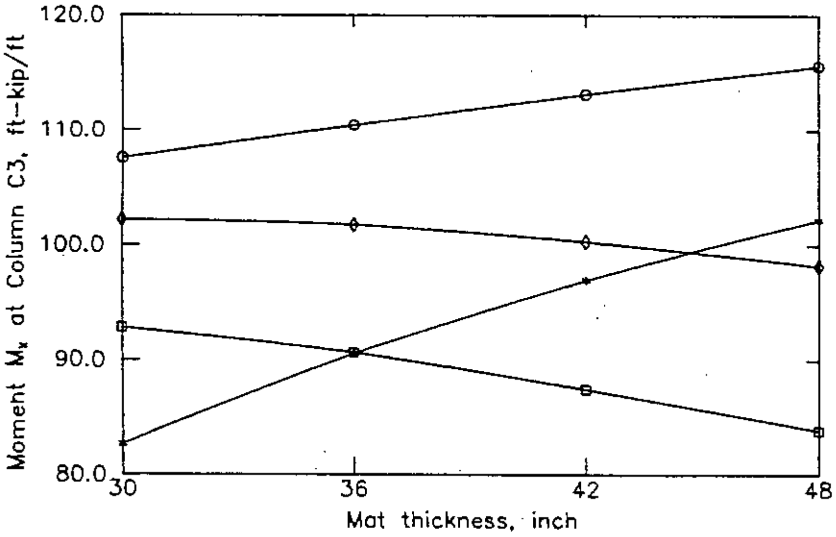


Fig.5.26 Variation of moment M_x at column C3 with mat thickness

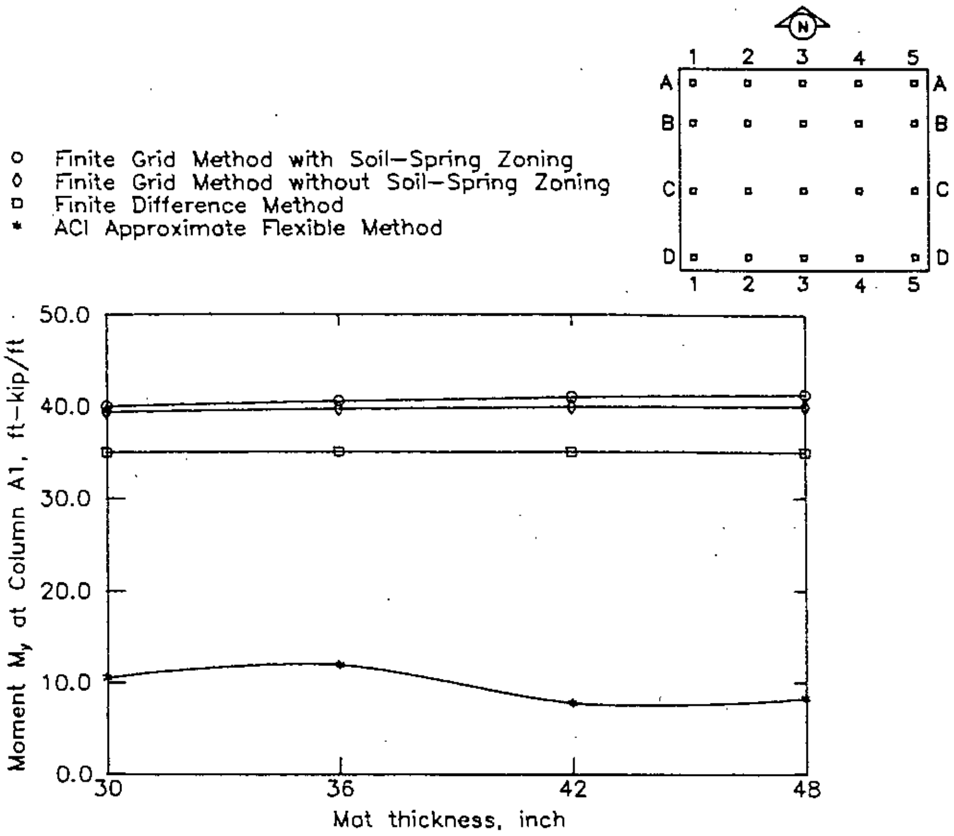


Fig.5.27 Variation of moment M_y at column A1 with mat thickness

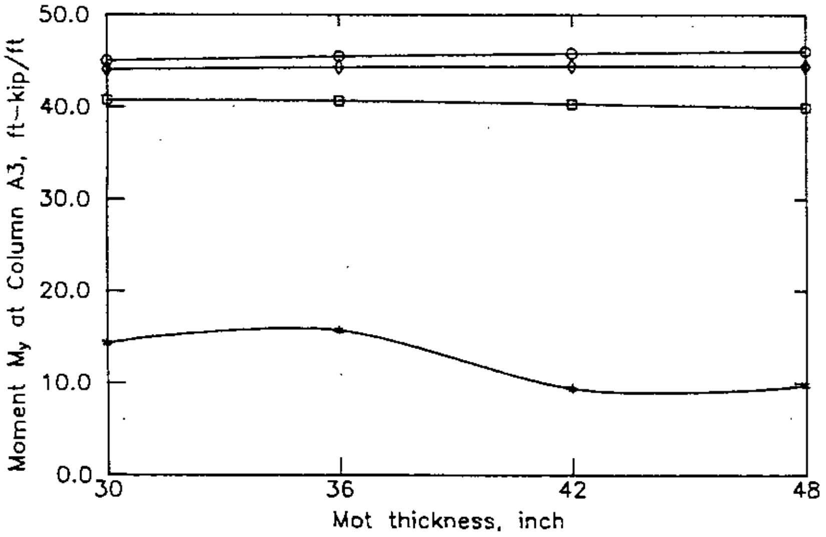


Fig.5.28 Variation of moment M_y at column A3 with mat thickness

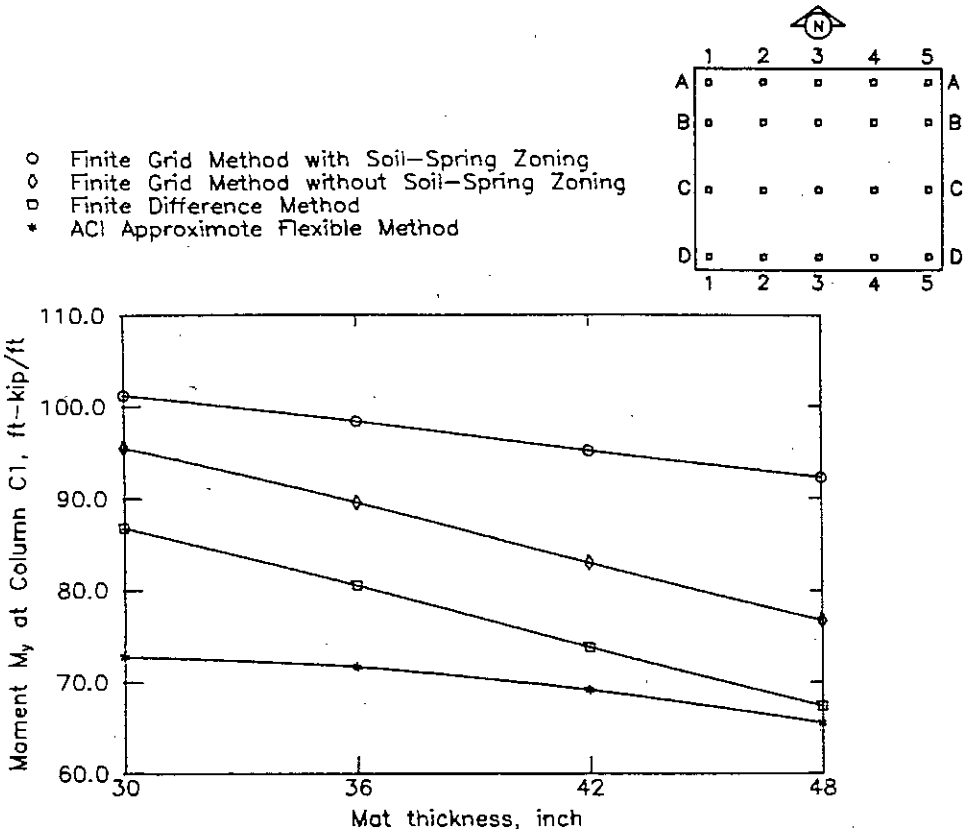


Fig.5.29 Varíation of moment M_y at column C1 with mat thickness

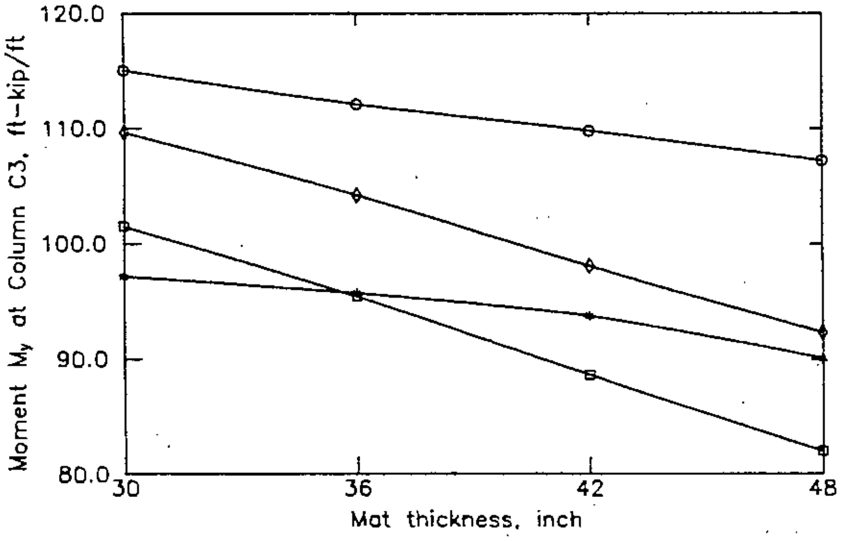


Fig.5.30 Variation of moment M_y at column C3 with mat thickness

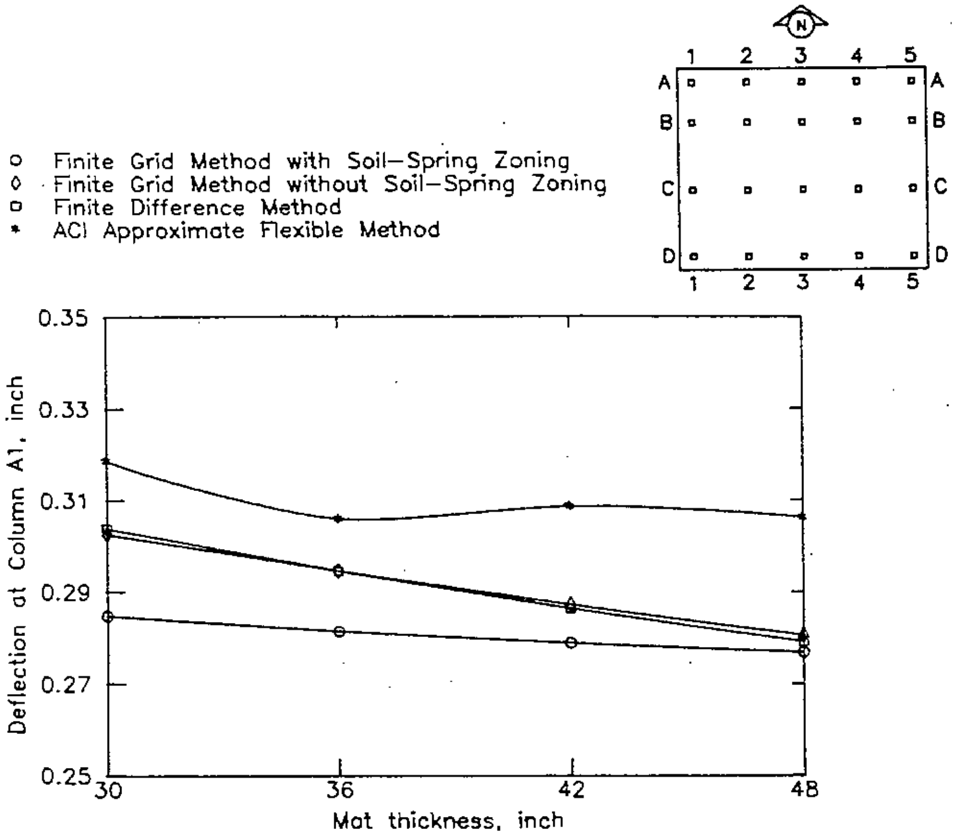


Fig.5.31 Variation of deflection at column A1 with mat thickness

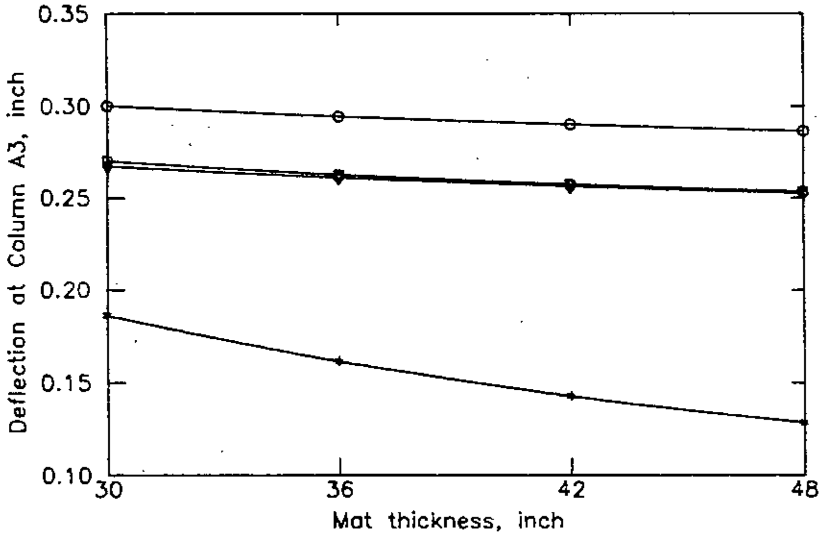


Fig.5.32 Variation of deflection at column A3 with mat thickness

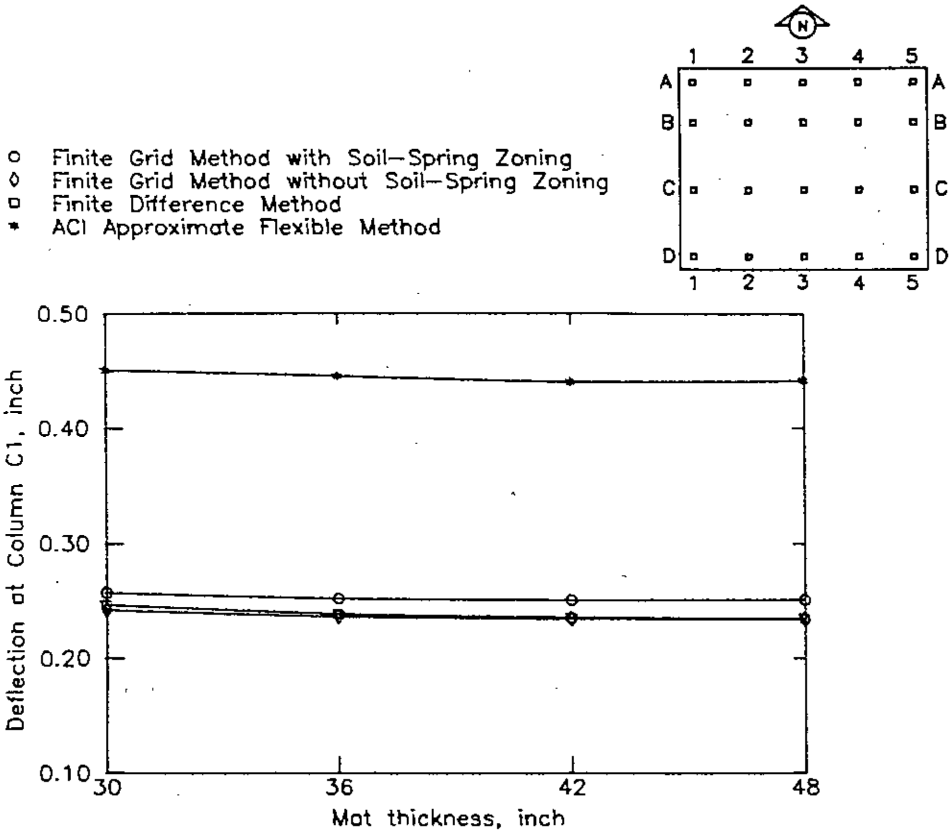


Fig.5.33 Variation of deflection at column C1 with mat thickness

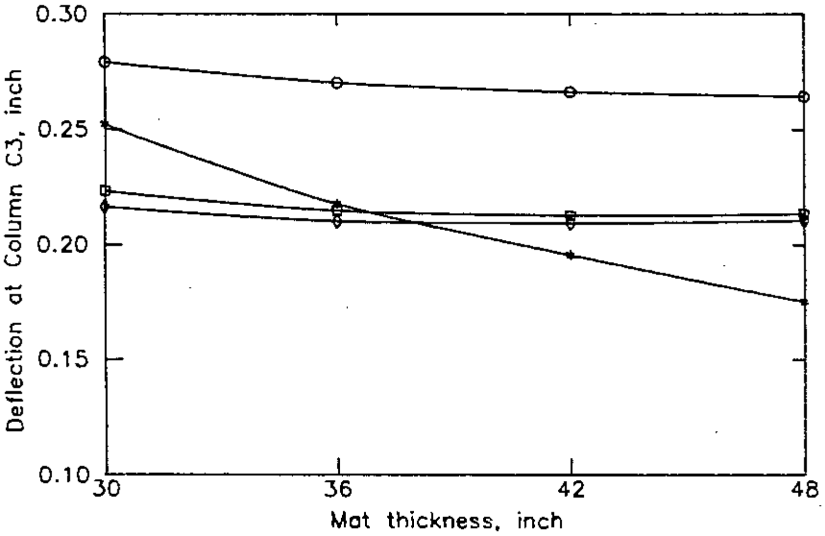


Fig.5.34 Variation of deflection at column C3 with mat thickness

5000 psi, keeping all other parameters constant. Increase in modulus of elasticity results in increased flexural stiffness. Consequently, a stiffer mat transfers load to soil more uniformly. Column moments may also be reduced with an increase in concrete strength. Moments of both directions at column points A1, A3, C1, C3 are considered in this study.

Moment M_x at column points A1, A3, C1 and C3 are presented in Fig.5.35 to 5.38. At column A1, there is very little (less than 1%) reduction in moment due to an increase in f'_c from 2500 to 5000 psi. At column point A3, moment obtained by both finite difference and finite grid method without soil spring zoning are 1.2% reduced for 5000 psi concrete in comparison to 2500 psi concrete. But finite grid with zoning moments are 1.7% higher for 5000 psi concrete. ACI method does not yield comparable results.

At column C1, moment M_x obtained by different methods are slightly reduced for an increase in f'_c. At column C3, moment M_x is reduced by 1.1% and 0.5%, respectively, for finite difference and finite grid method without soil spring zoning. With soil spring zoning, finite grid method yields 1.2% higher moment for 5000 psi concrete than f'_c equal to 2500 psi. ACI method gives moments which are about 10 to 20% lesser than finite grid method with soil spring zoning, but here the moment increases with an increase in f'_c.

Moment M_y at column points at A1, A3, C1 and C3 are presented in Fig.5.39 to 5.42. At column point A1 and A3, the moments are more or less unchanged with the change in f'_c. At column point C1 and C3, all the methods show a reduction in moment with an increase in f'_c from 2500 to 5000 psi. At C1, these reductions for finite grid with and without zoning and finite difference method are 1.9%, 3.8% and 4.0% respectively.

Effect on deflection (due to variation in f'_c)

The variation in deflection with f'_c at column point A1, A3, C1, C3 are presented in Fig.5.43 to 5.46. At all the column points, deflections obtained by all the methods indicate reduction due to increase in f'_c from 2500 to 5000 psi. This reductions at column A1 for finite grid with and without zoning, finite difference and ACI method are 0.8%, 1.6%, 1.9% and 0.5% respectively. The same reduction for A3 are 1.2%, 1.5%, 1.9% and 9.1%, respectively. For C1 and C3 they are 1.7%, 2.0%, 2.6%, 1.1% and 2.5%, 2.6%, 3.2%, 9.1%,

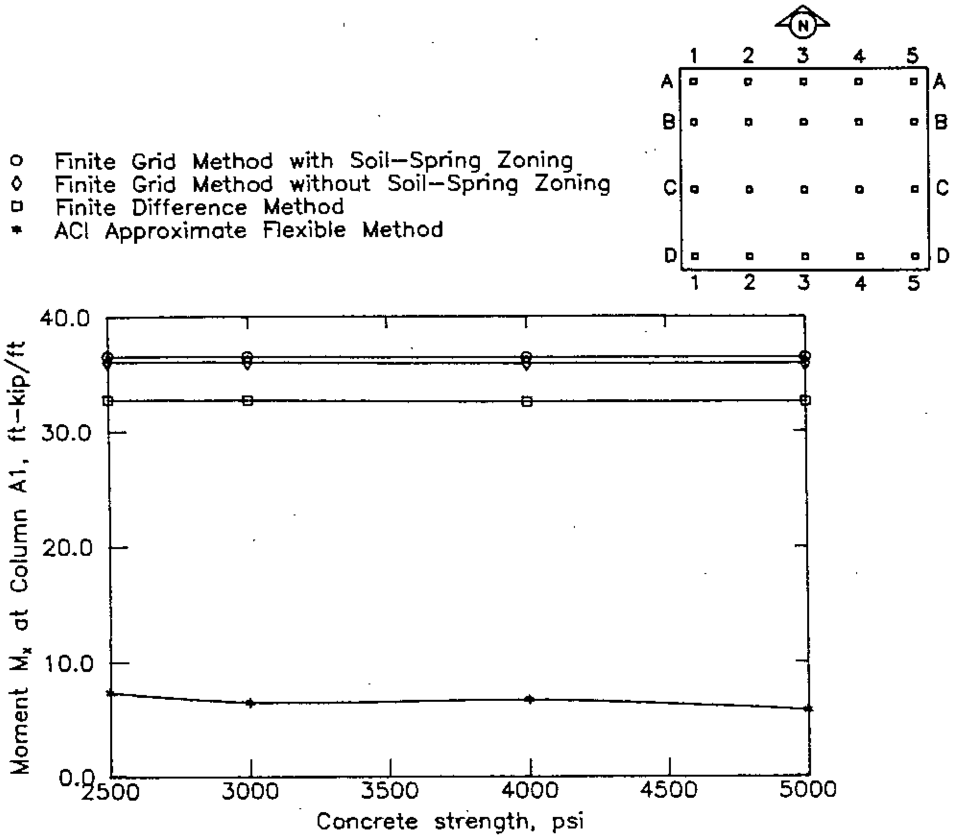


Fig.5.35 Variation of moment M_x at column A1 with concrete strength

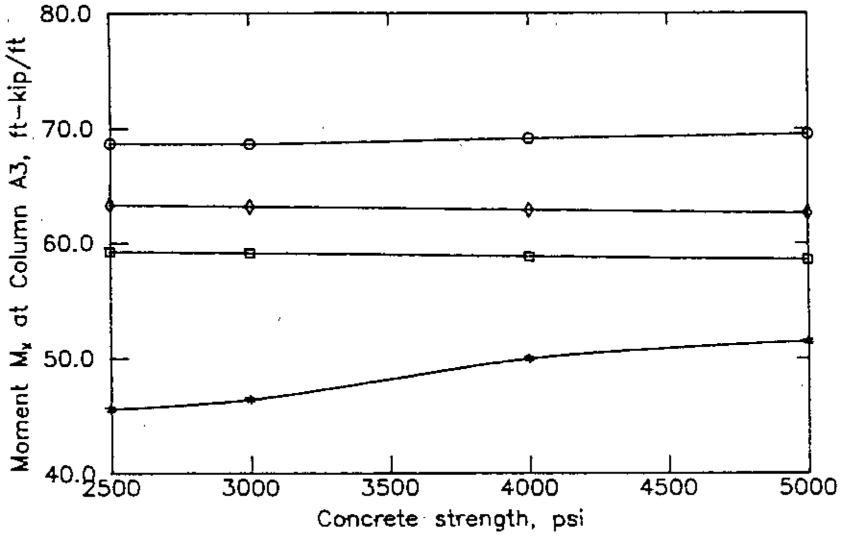


Fig.5.36 Variation of moment M_x at column A3 with concrete strength

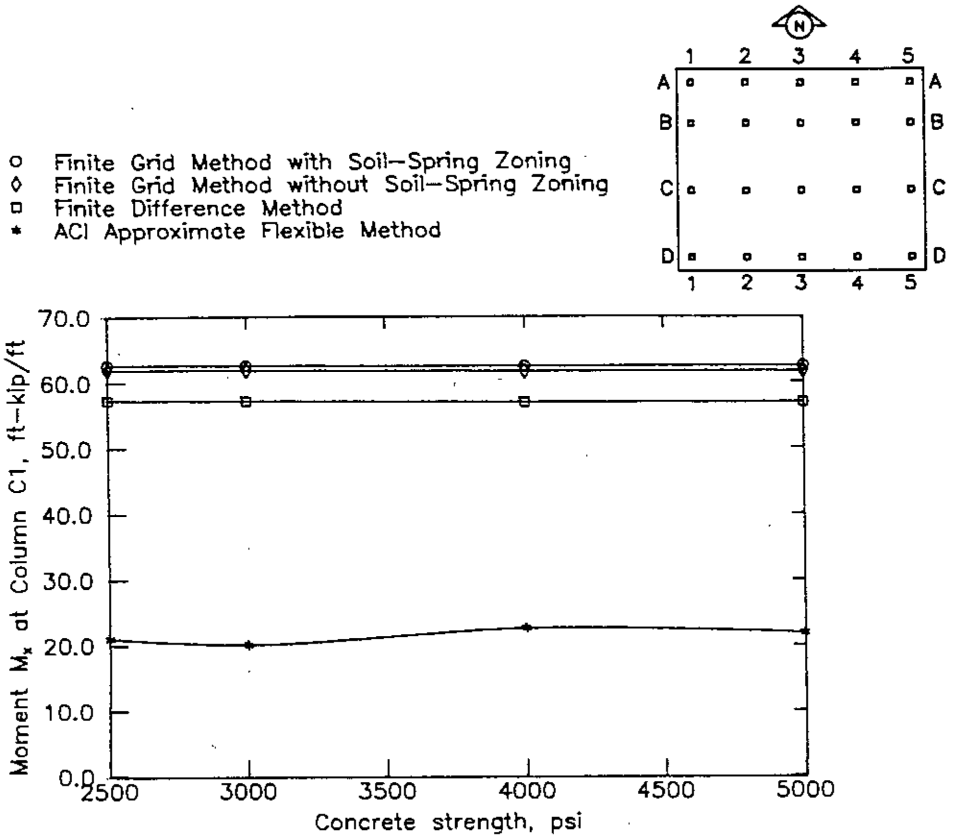


Fig.5.37 Variation of moment M_x at column C1 with concrete strength

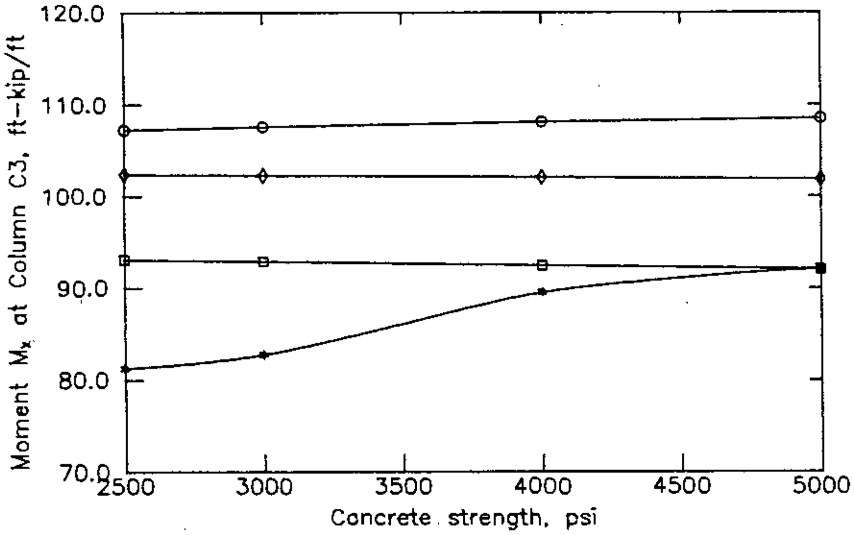


Fig.5.38 Variation of moment M_x at column C3 with concrete strength

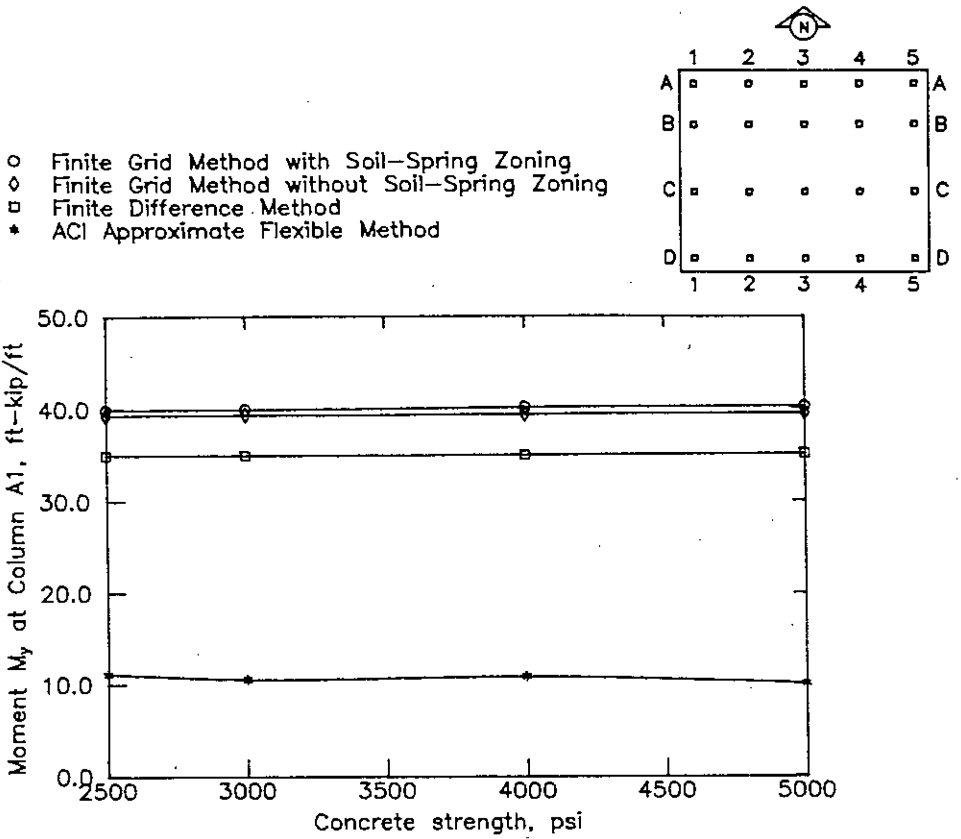


Fig.5.39 Variation of moment M_y at column A1 with concrete strength

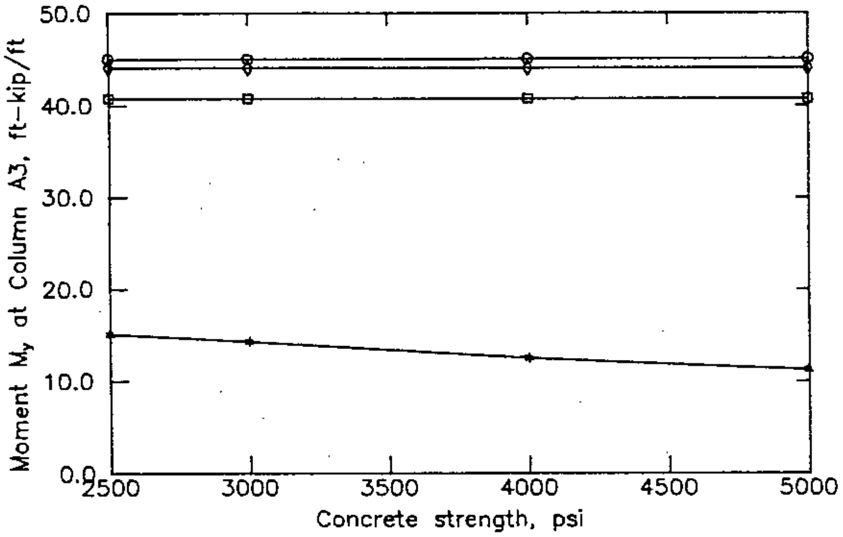


Fig.5.40 Variation of moment M_y at column A3 with concrete strength

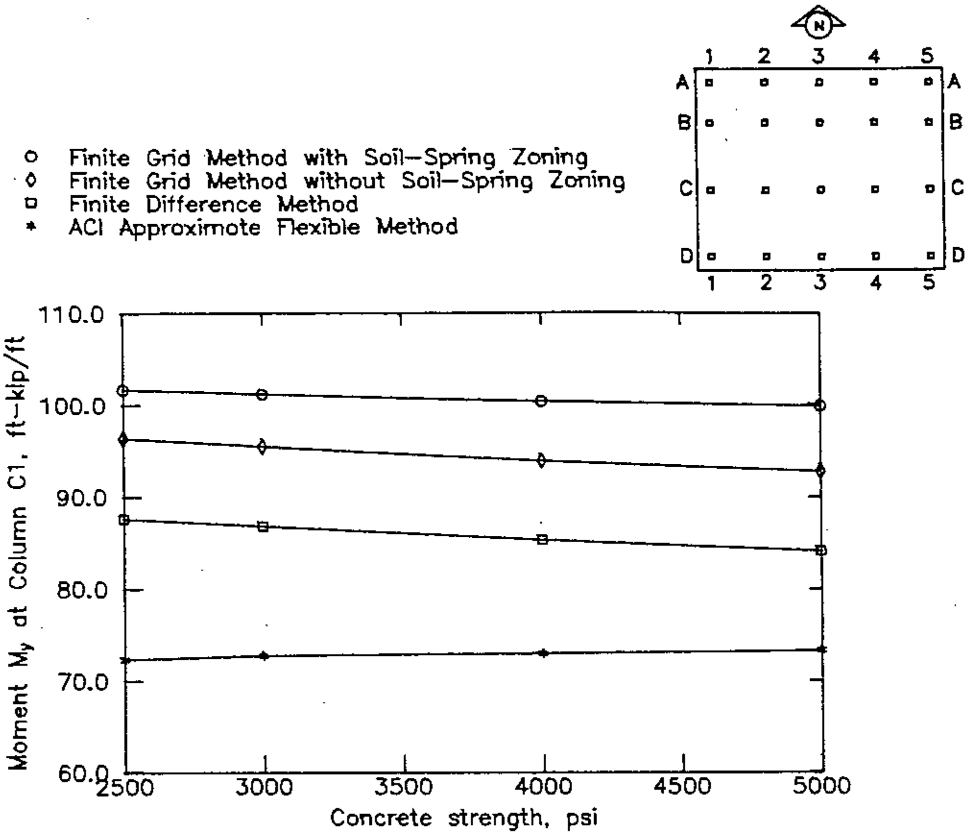


Fig.5.41 Voriotion of moment M_y at column C1 with concrete strength

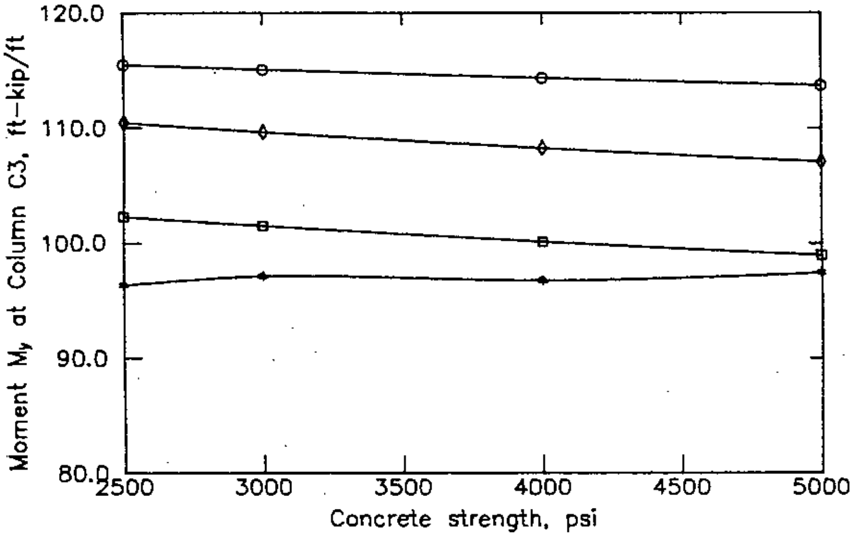


Fig.5.42 Voriation of moment M_y at column C3 with concrete strength

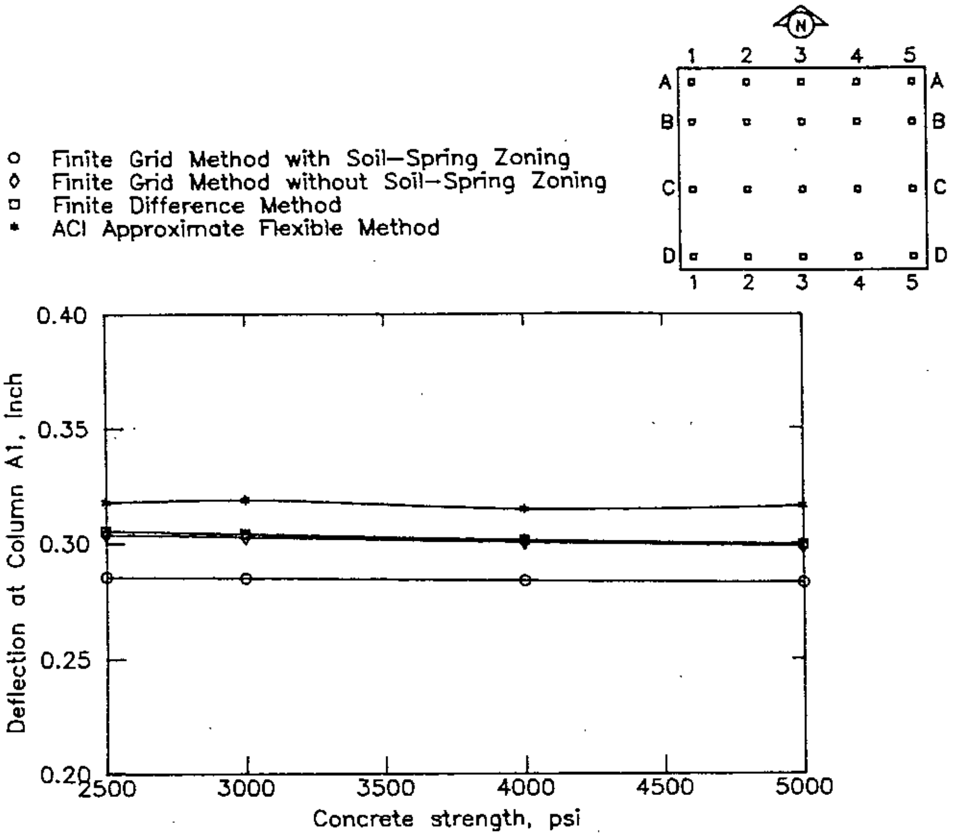


Fig.5.43 Variation of deflection at column A1 with concrete strength

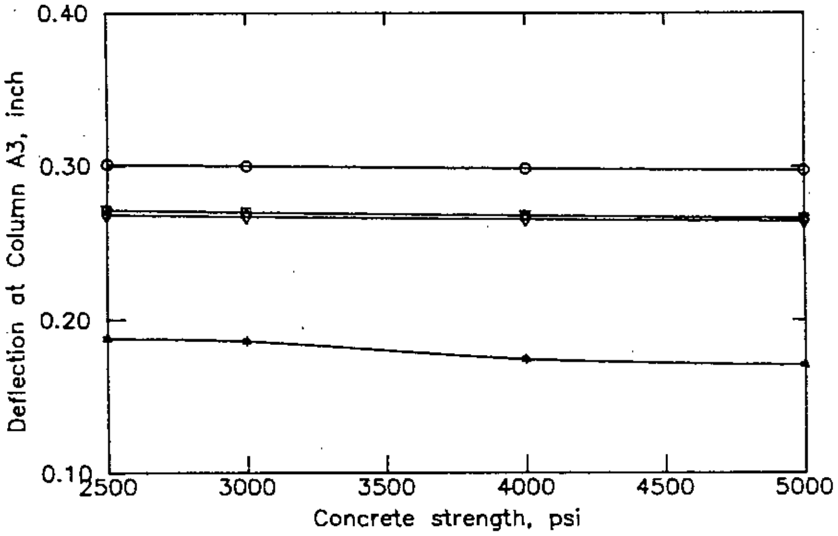


Fig.5.44 Variation of deflection at column A3 with concrete strength

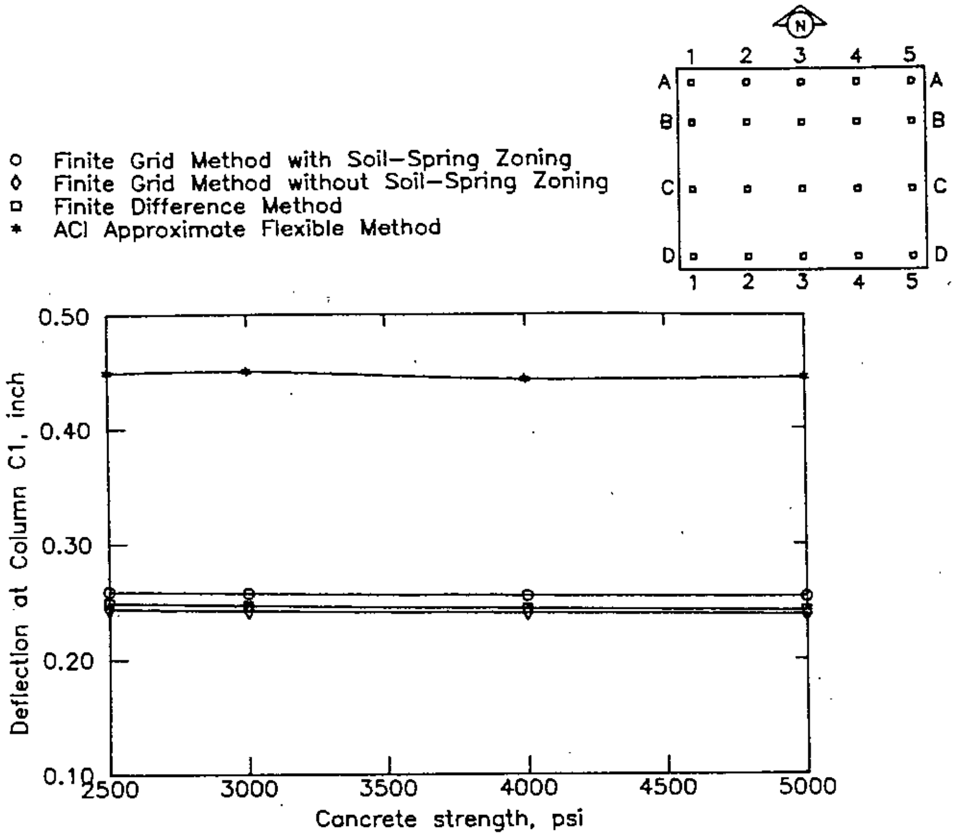


Fig.5.45 Variation of deflection at column C1 with concrete strength

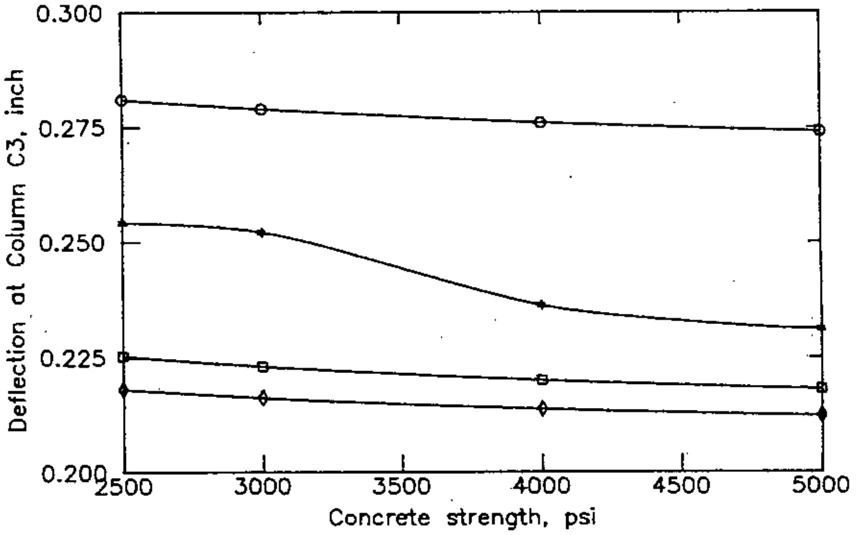


Fig.5.46 Variation of deflection at column C3 with concrete strength

respectively.

It is worth noticing that the effect of variation in mat thickness and concrete strength is similar, since both the parameters actually modify the overall stiffness of the mat.

5.4.3 Effect of Poisson's Ratio

Effect on moment

To find the effect of Poisson's ratio (μ) on moment, μ is varied from 0.15 to 0.25, which is the usual range of concrete. Four points A1, A3, C1, C3 are selected where moments in both directions are to be considered. Shukla (1984) stated that higher μ gives higher moments. This is true for the method he discussed i.e. ACI approximate flexible method because higher μ value results in higher moment coefficients and also this is valid for finite difference method. But in finite grid method, where beam-column elements are used, μ is only used to calculate shear modulus of concrete, which is very slightly affected by μ.

The moment M_x at column point A1, A3, C1, C3 with different μ values are plotted in Fig.5.47 to 5.50. Moments obtained by finite grid method with or without zoning show very little increase due to an increase in μ. Finite difference method yields moments (at A1, A3, C1, C3) 6.8%, 5.3%, 12.7% and 9.4% higher for a μ of 0.25 than for a μ of 0.15. The same increase for ACI method are 23.5%, 17.6%, 30.5%, and 16.9% respectively.

The variation of moment M_y at column A1, A3, C1, C3 with μ is presented in Fig.5.51 to 5.54. At A1, A3, C1 and C3, moments obtained by finite difference method for a μ of 0.25 is higher by 8.3%, 11.9%, 4.8% and 7.5% respectively than the same for a μ of 0.15. The same increase for ACI method is 31.7%, 26.3%, 13.6% and 11.9%, respectively. Finite grid method with or without zoning shows little increase in moment, as it should.

Effect on deflection

Poisson's ratio has very little effect on deflection for all the methods. Deflections of column points A1, A3, C1, C3 are presented in Fig.5.55 to 5.58.

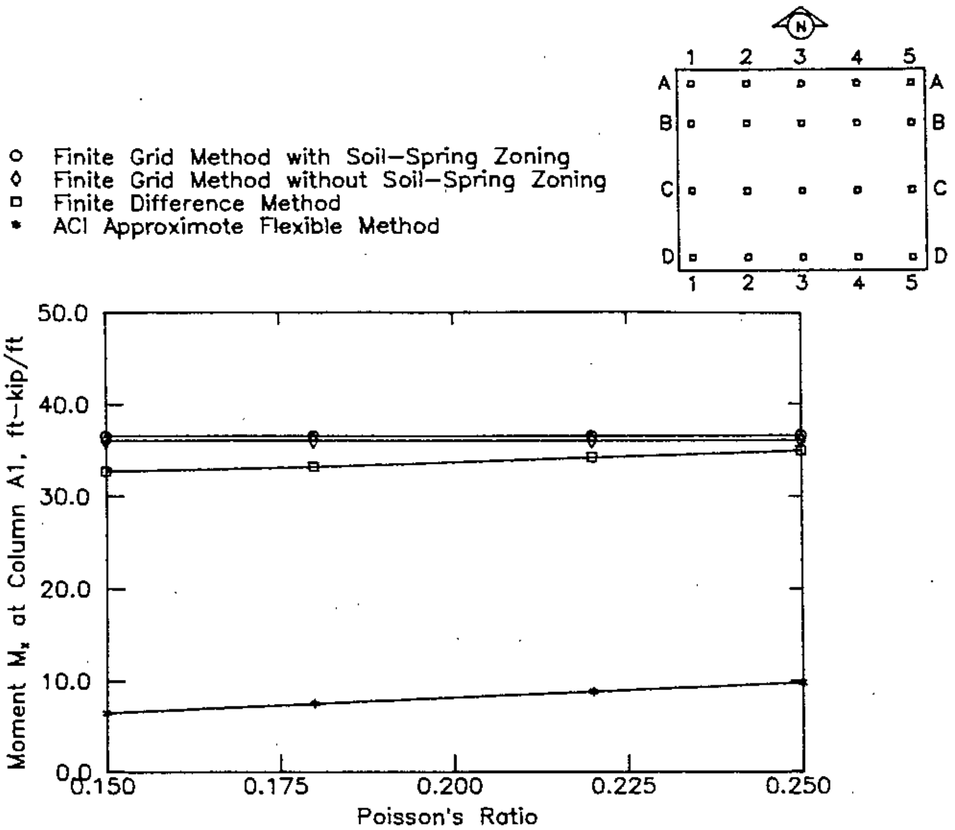


Fig.5.47 Variation of moment M_x at column A1 with Poisson's ratio

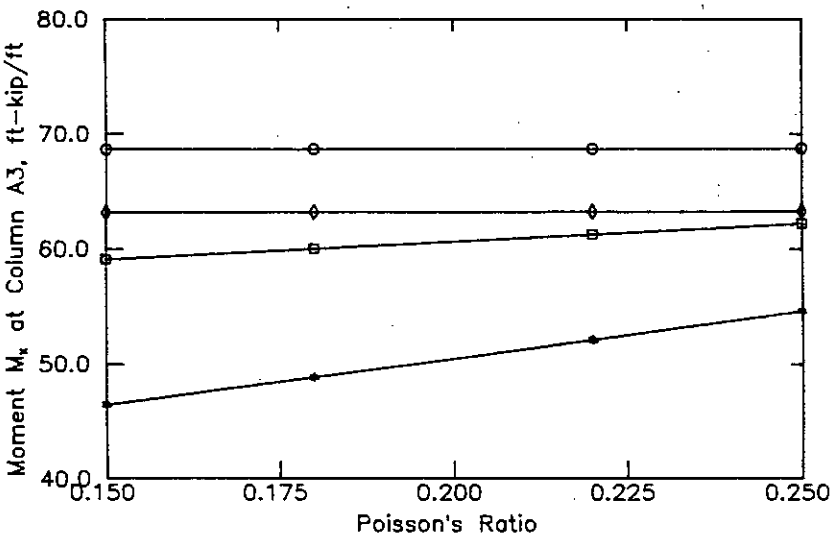


Fig.5.48 Variation of moment M_x at column A3 with Poisson's ratio

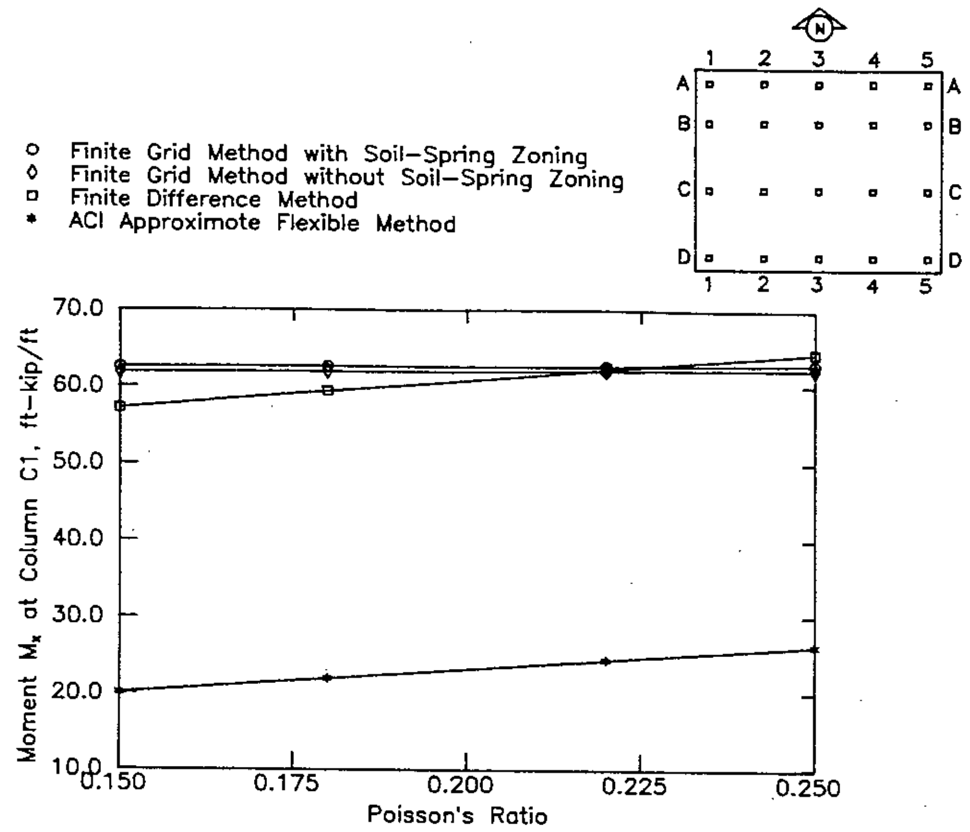


Fig.5.49 Variation of moment M_x at column C1 with Poisson's ratio

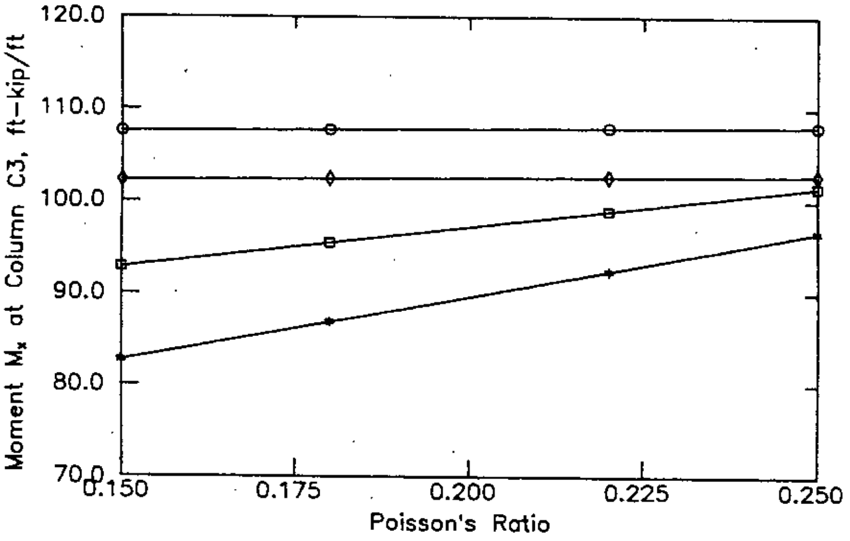


Fig.5.50 Variation of moment M_x at column C3 with Poisson's ratio

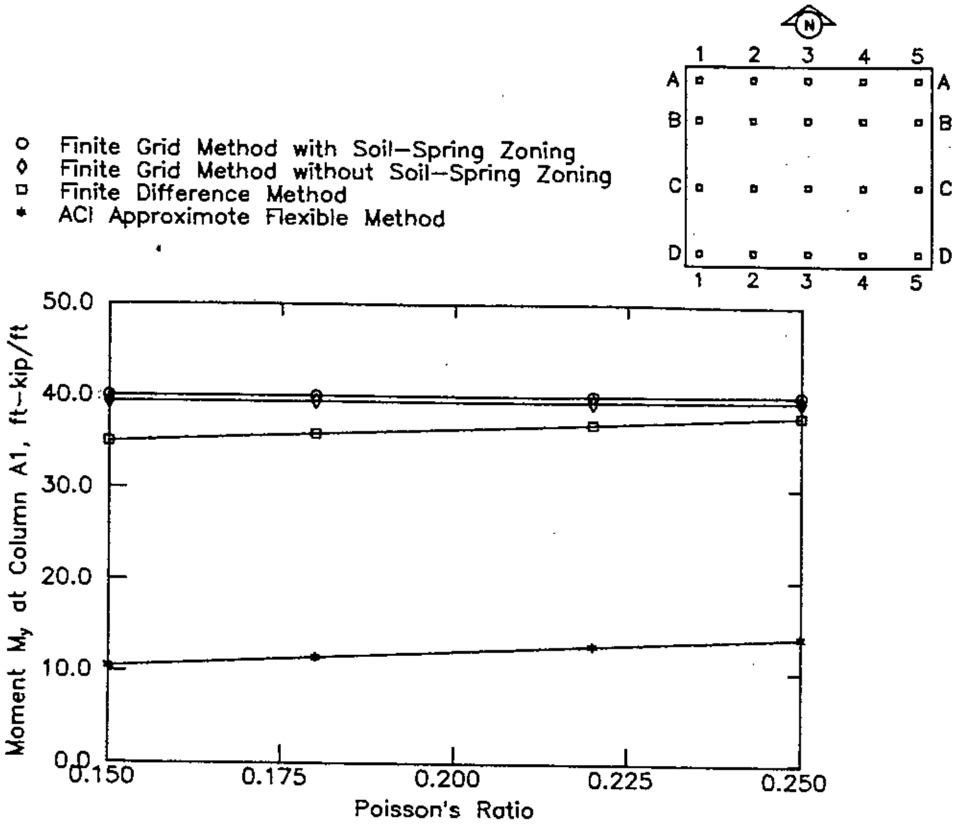


Fig.5.51 Voriation of moment M_y at column A1 with Poisson's rotio

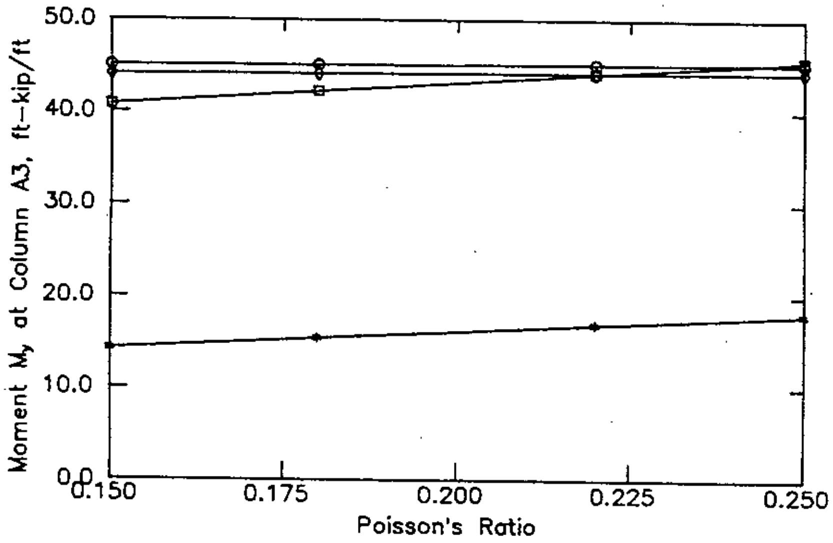


Fig.5.52 Voriation of moment M_y at column A3 with Poisson's rotio

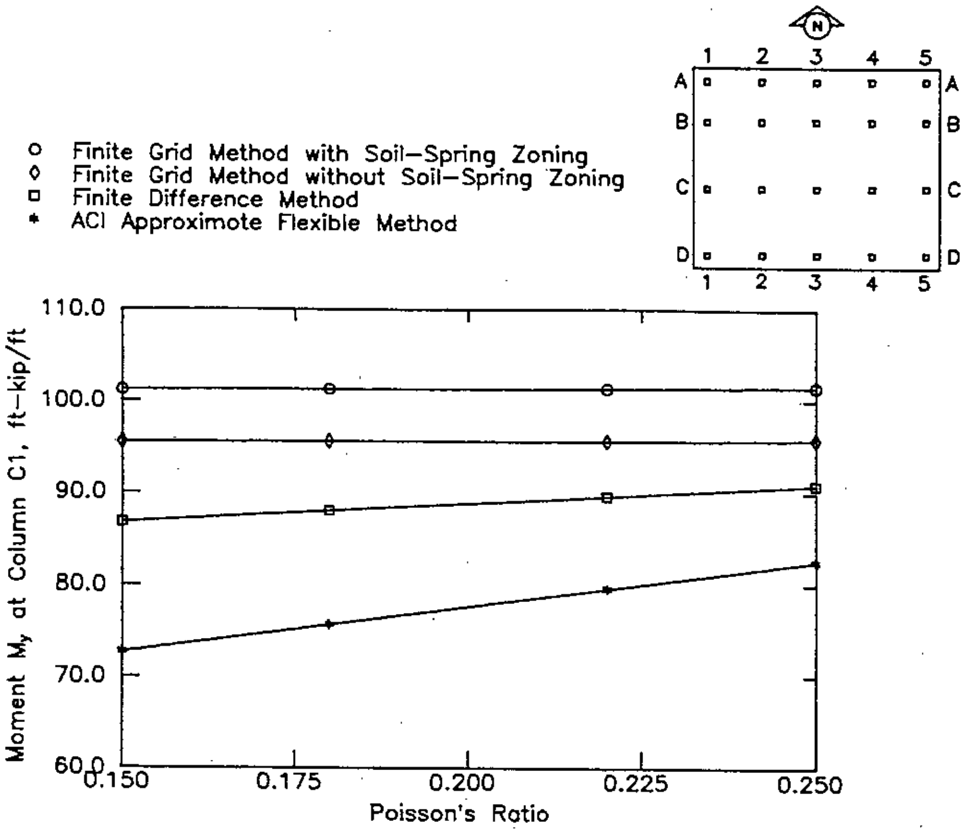


Fig.5.53 Variation of moment M_y at column C1 with Poisson's ratio

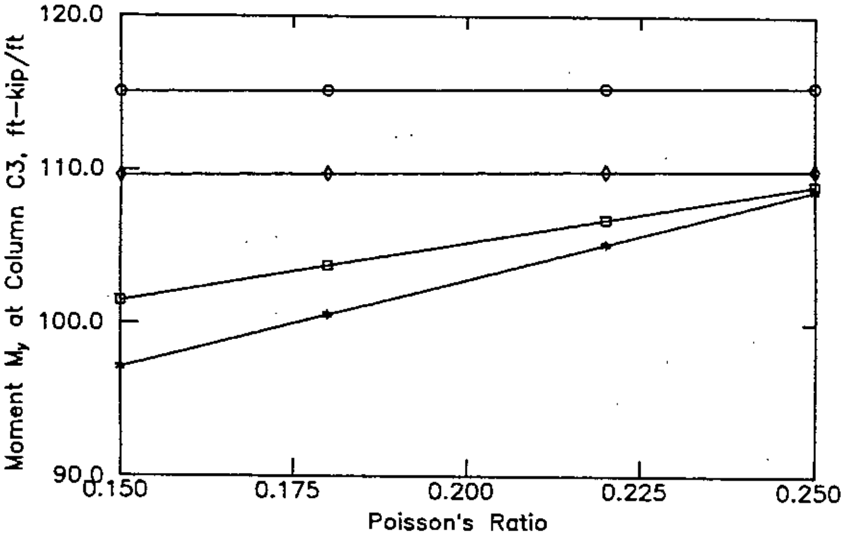


Fig.5.54 Variation of moment M_y at column C3 with Poisson's ratio

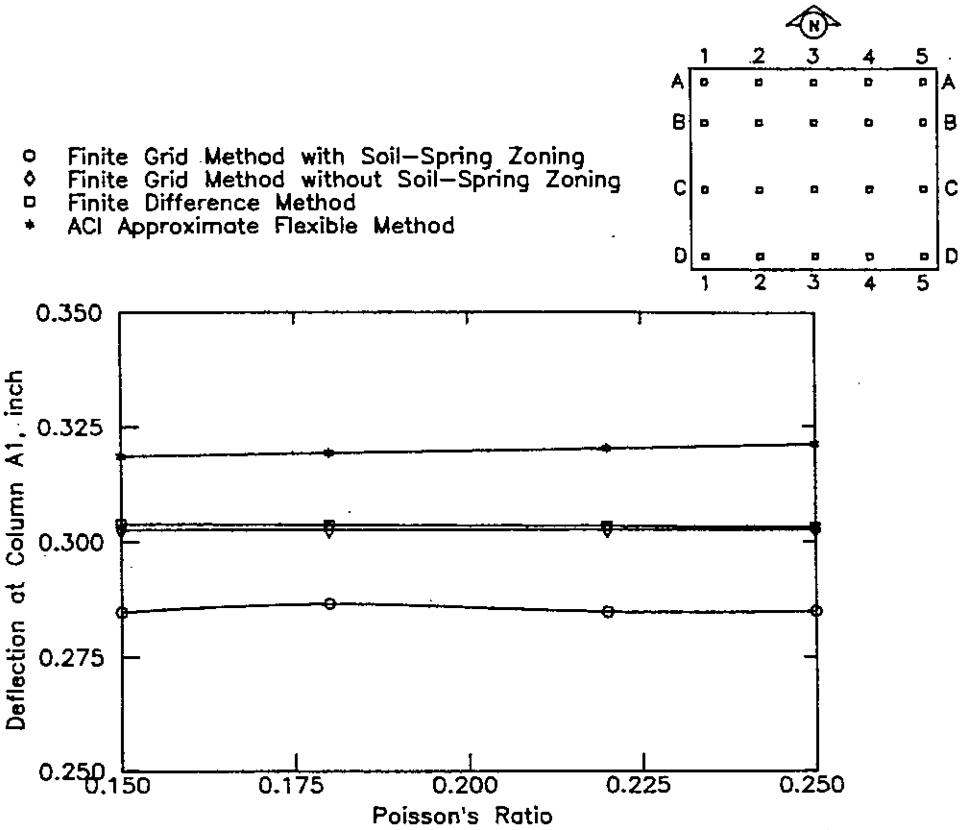


Fig.5.55 Variation of deflection at column A1 with Poisson's ratio

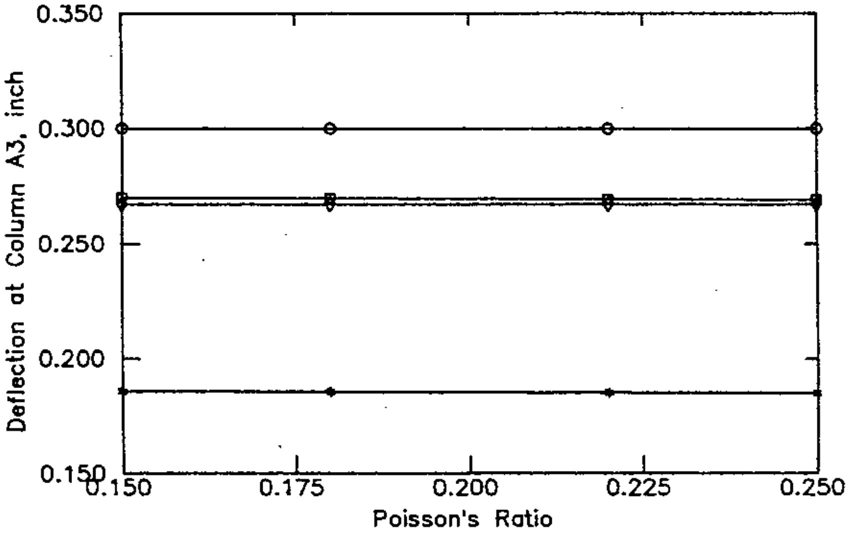


Fig.5.56 Variation of deflection at column A3 with Poisson's ratio

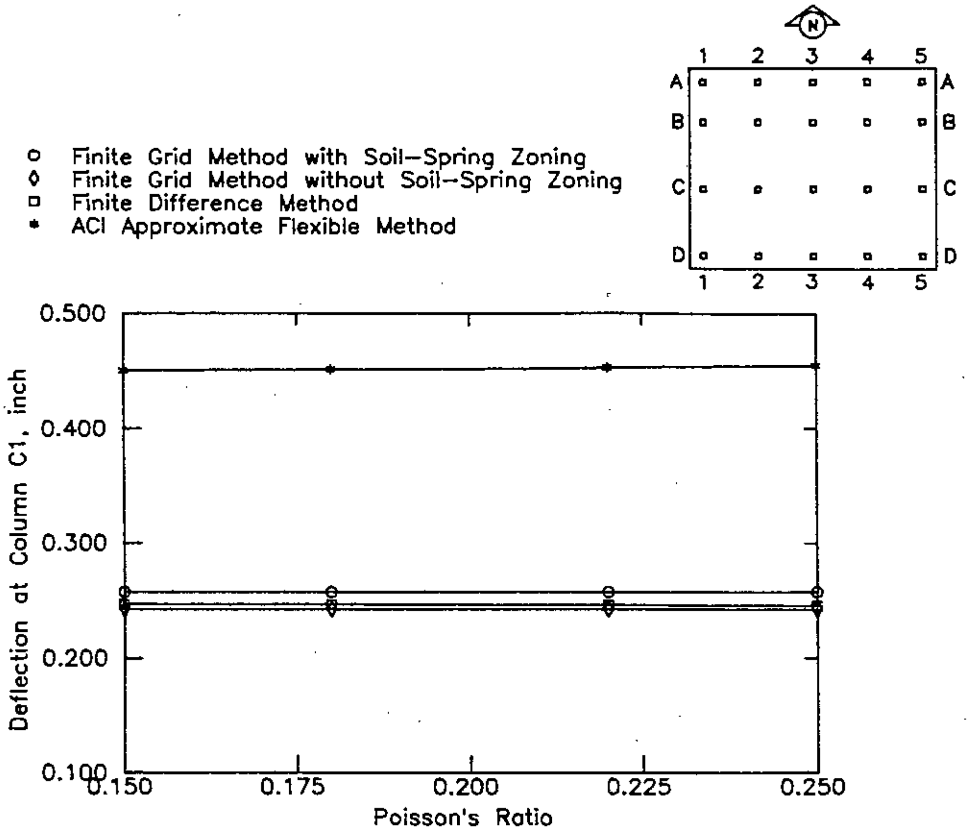


Fig.5.57 Variation of deflection at column C1 with Poisson's ratio

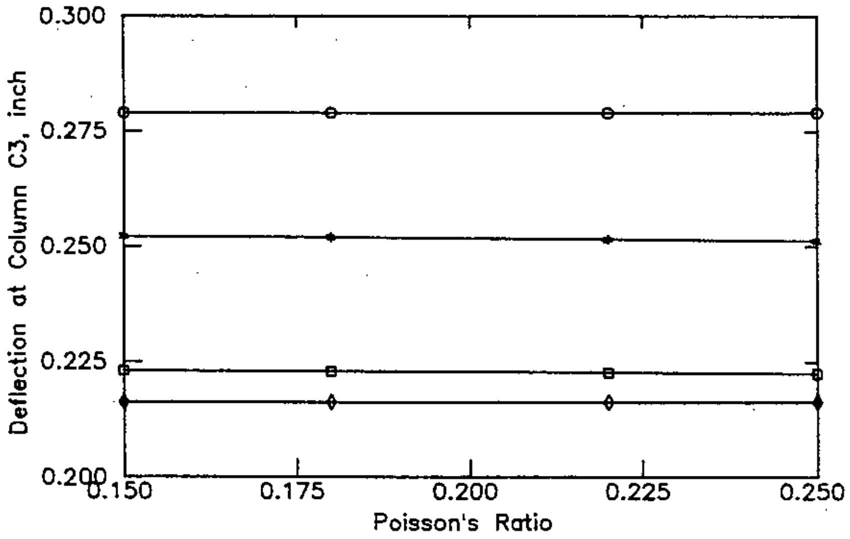


Fig.5.58 Variation of deflection at column C3 with Poisson's ratio

Finite difference method shows very small (less than 1%) decrease in deflection for a μ of 0.25 compared to a μ of 0.15. Similar findings are obtained for ACI method. Finite grid method with or without zoning, on the other hand shows no change in deflection for a increase in μ.

5.4.4 Effect of Loads

To determine the effect of vertical and horizontal loads and their combination, a 14 storied building is considered having the same plan as used in the earlier investigation. The dead load and live load on each column is calculated and the design vertical load is found to be nearly equal to twice the load for a 7 storied building. Only finite grid method with soil spring zoning is employed in the analysis.

The bending moments along column lines A and D for a 7 and a 14 storied buildings under only vertical load are presented in Fig.5.59 and 5.60. It is interesting to observe that doubling the loads keeping all other parameters constant, makes moments at all the points double and the deflections are also doubled.

To find the effect of wind load, three load cases are considered : (1) dead and live loads only, (2) dead, live and northward wind loads and (3) dead, live and southward wind loads. The bending moments along column line A with these load cases are shown in Fig.5.61. There is no effect of southward wind in this column line. Northward wind increases moment values at all the points. This increase is 69.5%, 41.4% and 41.0% for column A1, A2, A3, respectively.

The bending moments along column line D are presented in Fig.5.62. Along this south edge column line, there is no effect of northward wind. But southward wind results in higher moments at D1, D2 and D3, which are 51.3%, 32.6% and 32.4% higher than moment values if dead and live loads are only considered.

Bending moments along column line 1 and 3 are presented in Fig.5.63 and 5.64. Southward wind has no effect at A1, but with the increase in distance towards south, increased negative moments are found between columns. At column B1 and C1, column moments are reduced due to southward wind, but at D1 it is increased by 53.0%. North wind has similar effects on the reverse

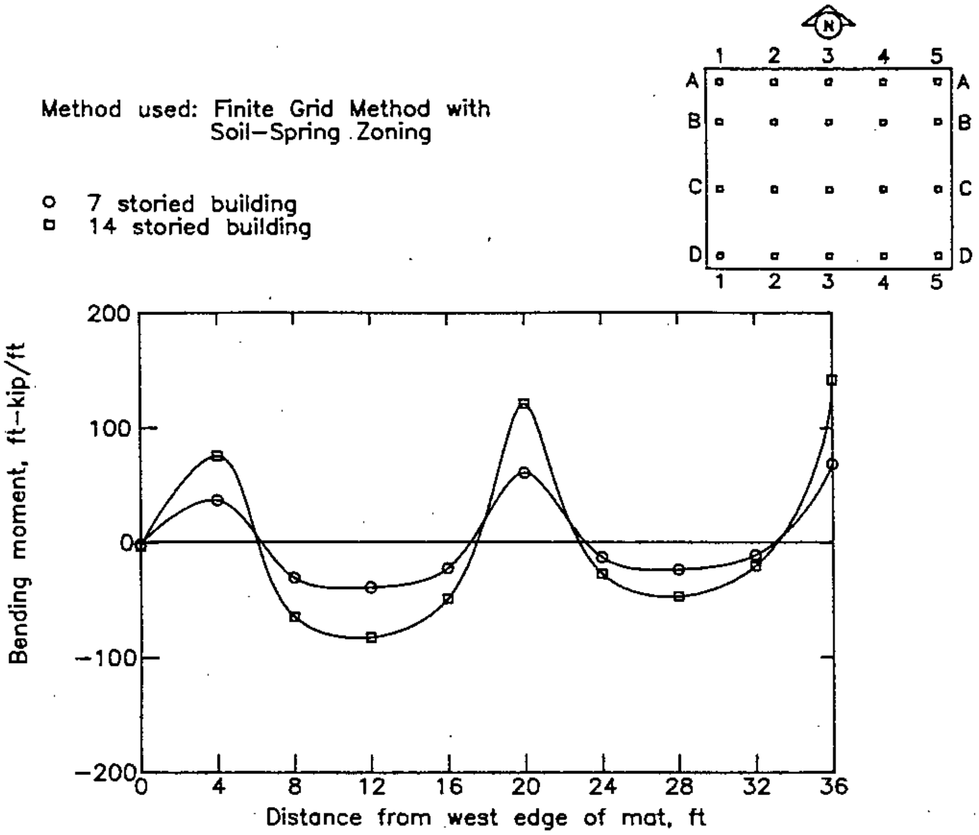


Fig.5.59 Variation of moment along column line A with variation in vertical loads

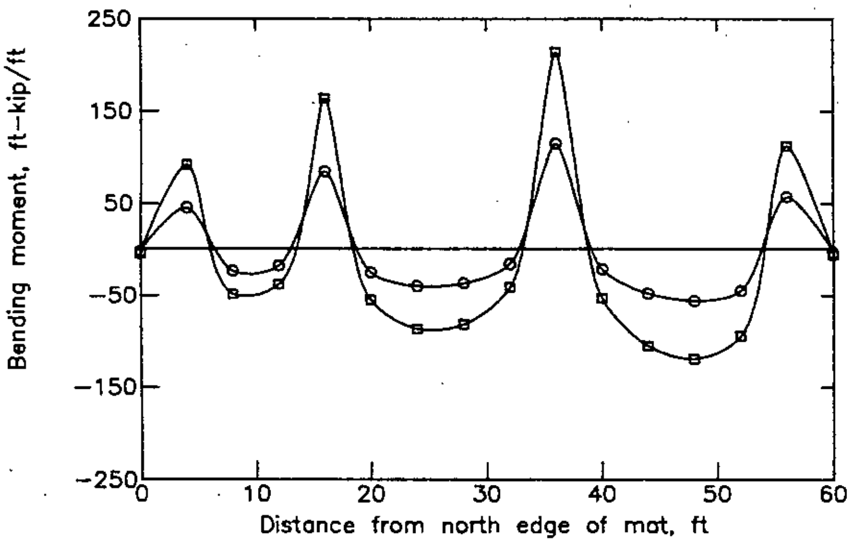


Fig.5.60 Variation of moment along column line 3 with variation in vertical loads

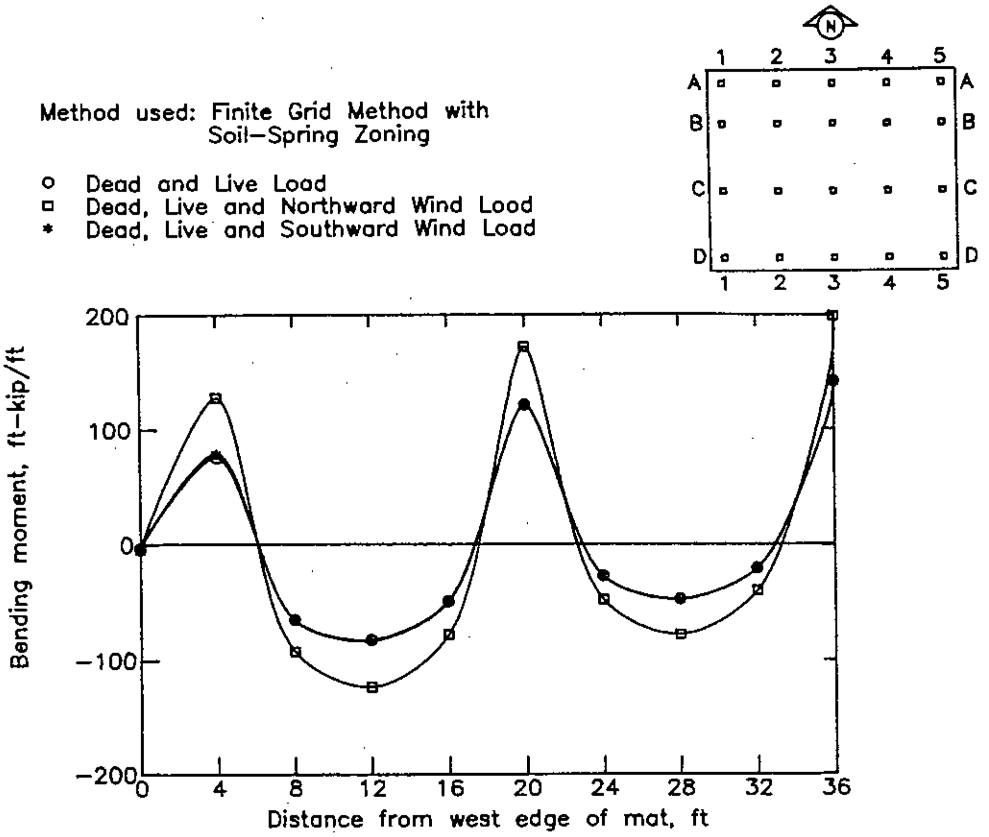


Fig.5.61 Variation of moment along column line A with different load cases

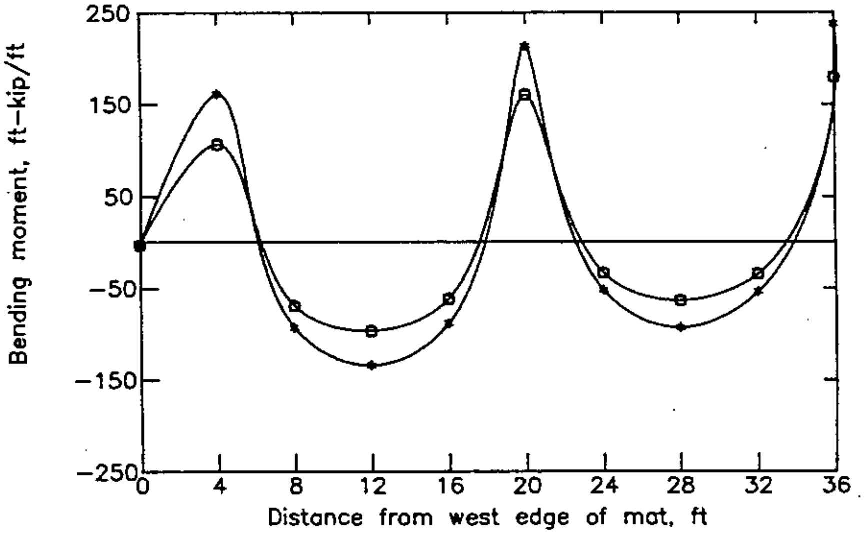


Fig.5.62 Variation of moment along column line D with different load cases

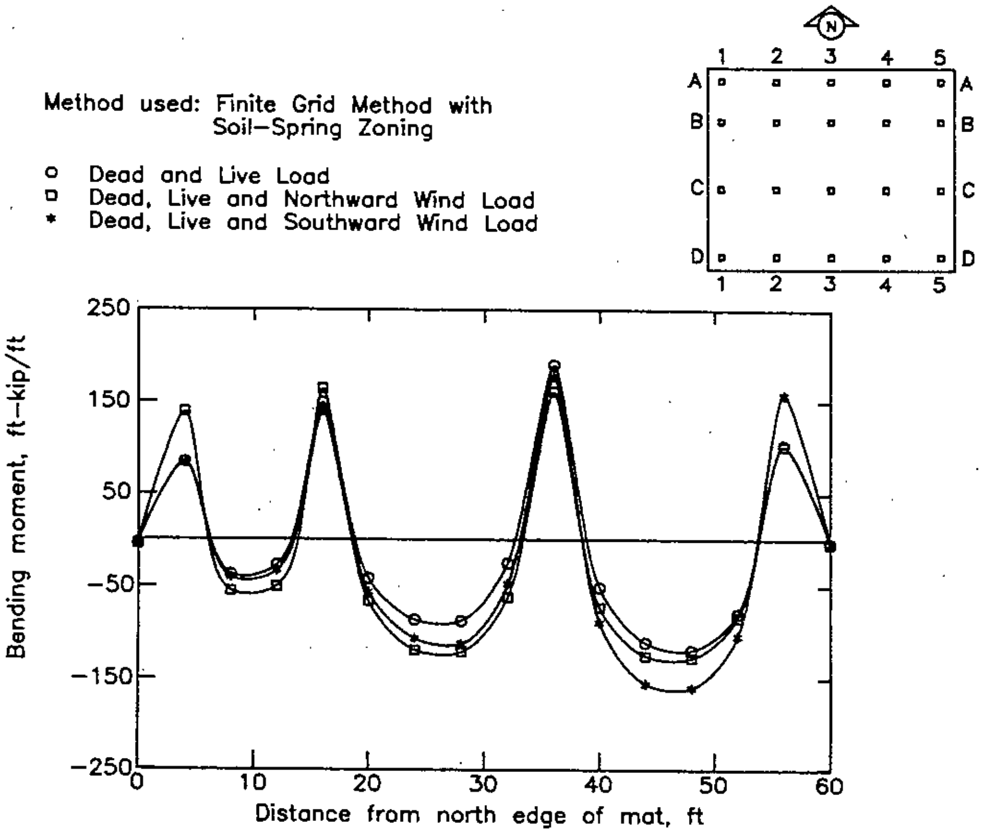


Fig.5.63 Variation of moment along column line 1 with different load cases

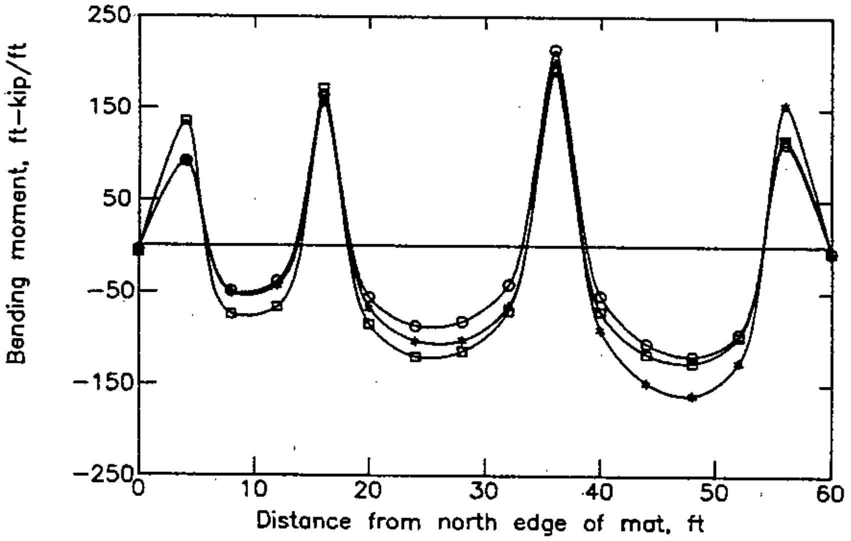


Fig.5.64 Variation of moment along column line 3 with different load cases

directions but here both A1 and B1 moments are increased. Bending moments along column line 3 yields similar results.

Deflections along column line A and D are presented in Fig.5.65 and 5.66. Due to northward wind, the column line A is more deflected compared to load case (1), but southward wind results uplift of column line compared to load case (1). Column line D deflects more for southward wind and less for northward wind when compared to load case (1).

The deflections along column lines 1 and 3 are shown in Fig.5.67 and 5.68. Northward wind causes the north side of mat to deflect more and, as a result, south side is slightly uplifted. South wind causes south edge of mat to deflect more and, at the same time, north edge deflections, found from load case (1), are slightly reduced.

5.4.5 Effect of Number and Spacing of Columns

To find the effect of column numbers, the same mat-soil system with 7 stories as used in the comparative study is taken. Keeping the plan area and the load same, the 5 columns along column line B are dropped and column line C is shifted 4 ft towards north to have a reasonable column spacing.

The bending moments along column line C with and without columns along line B are shown in Fig.5.69. It is to be noted that the spans along C remain unchanged but the column loads are 1.3 times increased as the contributing area of each column is 1.3 times more than before. The bending moment along line C is also 1.3 times increased when columns in line B are dropped.

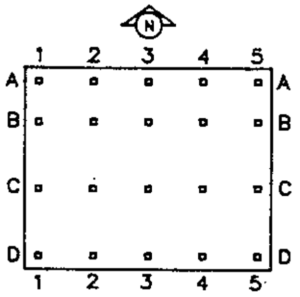
It is also interesting to note that the summation of bending moments along east-west direction remains almost the same with and without columns in B line.

The bending moments along column line 3 with and without column in line B are presented in Fig.5.70. Here column moments at A3, C3 and D3 are increased by 73.0%, 54.2% and 18.5% when columns in line B are dropped.

Moments are also compared by dropping column lines 2 and 4. Bending moments along column line 3 with and without columns in lines 2 and 4 are shown in Fig.5.71. By dropping column lines 2 and 4, contributing area of

Method used: Finite Grid Method with
Soil–Spring Zoning

- ○ Dead and Live Load
- □ Dead, Live and Northward Wind Load
- * Dead, Live and Southward Wind Load



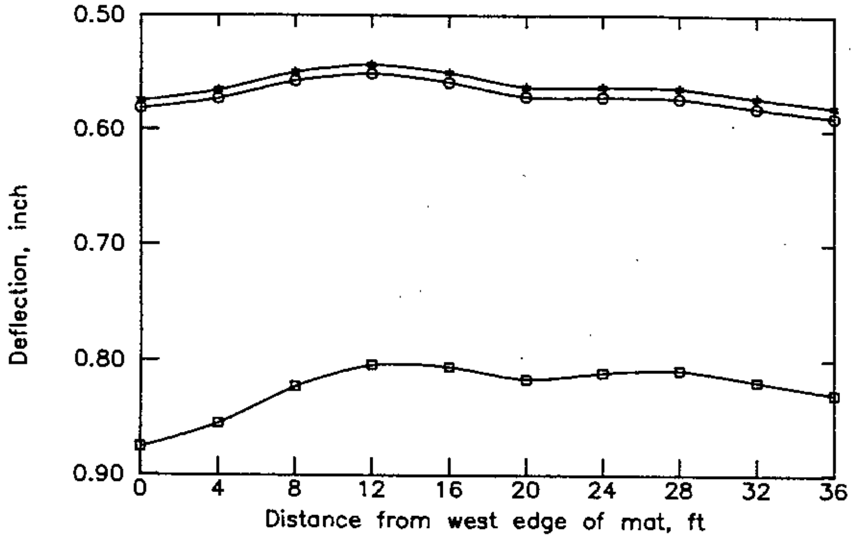


Fig.5.65 Variation of deflection along column line A with different load cases

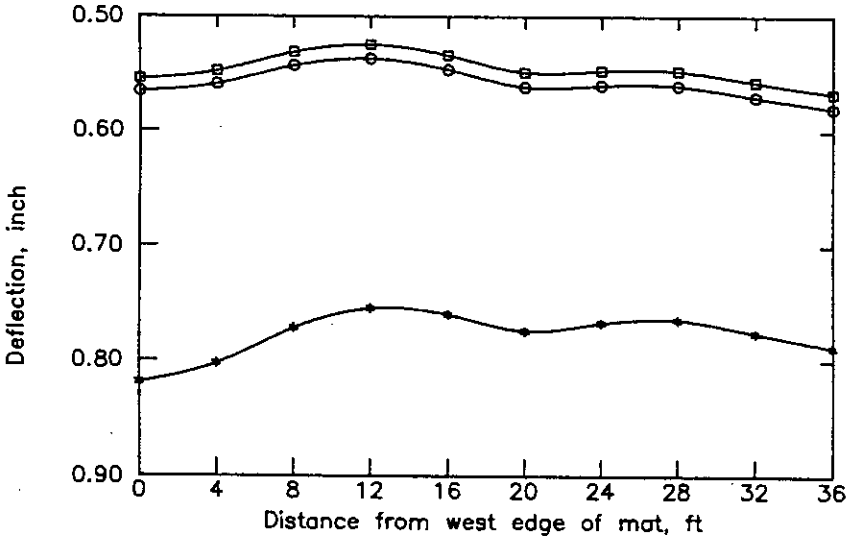


Fig.5.66 Variation of deflection along column line D with different load cases

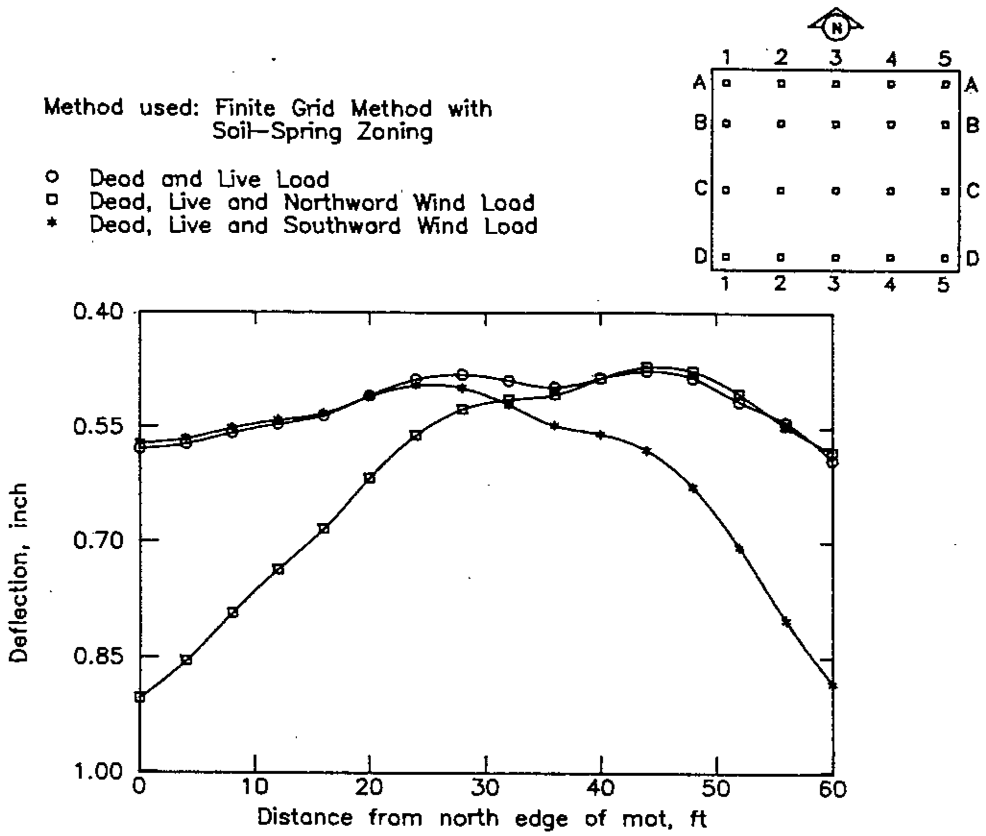


Fig.5.67 Variation of deflection along column line 1 with different load cases

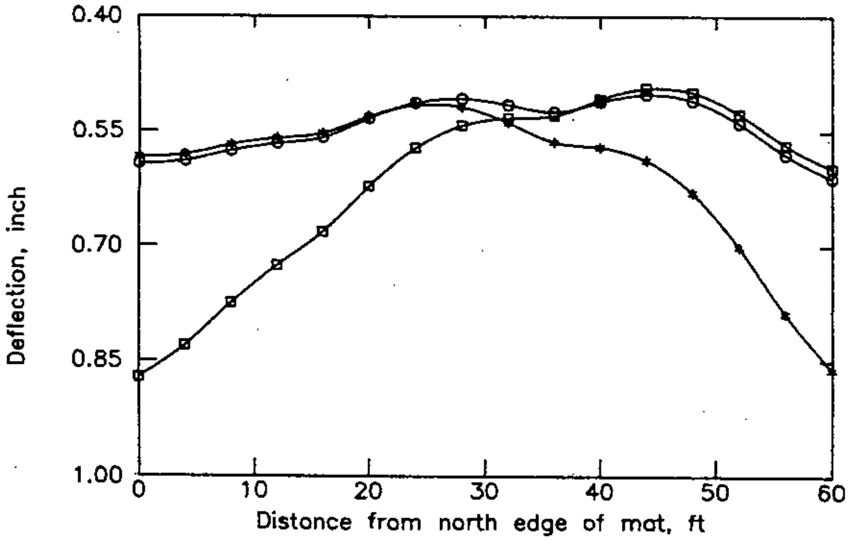


Fig.5.68 Variation of deflection along column line 3 with different load cases

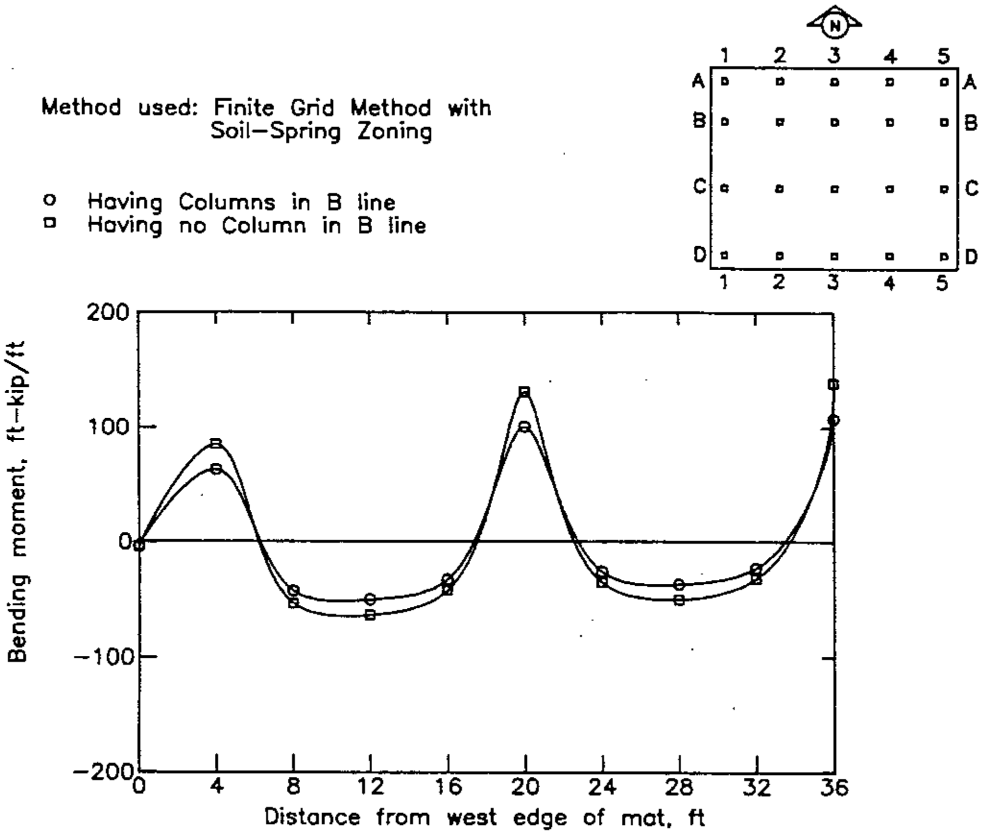


Fig.5.69 Variation of bending moment along column line C with and without columns in B line

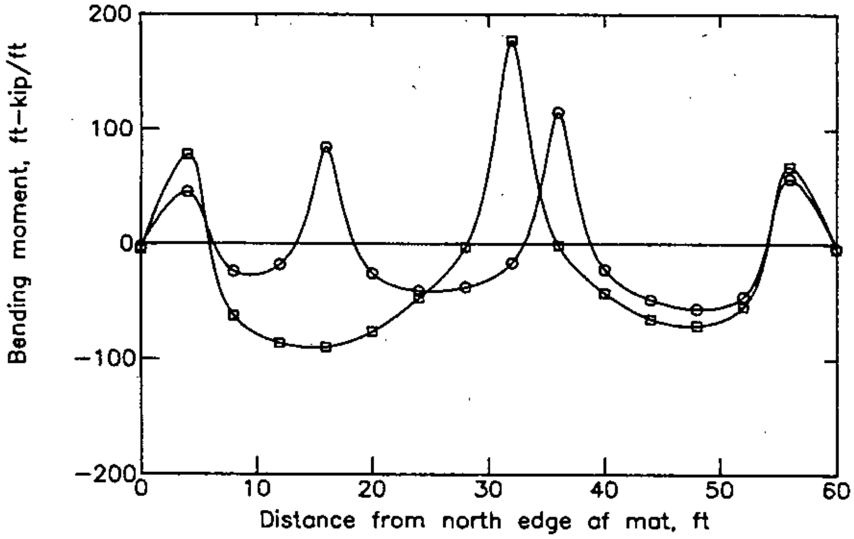


Fig.5.70 Variation of bending moment along column line 3 with and without columns in B line

column line 3 is doubled and thus, load is also doubled. It has been found that all the moments are doubled for dropping these columns. As earlier, the summation of all column moments in north-south direction remains same.

Moments along column line C are presented in Fig.5.72. Here column moments at C1, C3 are increased by 67.2%, 165.3% when columns in 2 and 4 lines are dropped.

It may be concluded that if spans remain unchanged in a direction of mat, the summation of column moments in that direction remains unchanged no matter how many parallel column lines are present. Total column moment in a direction is distributed among parallel column lines according to their contributing areas.

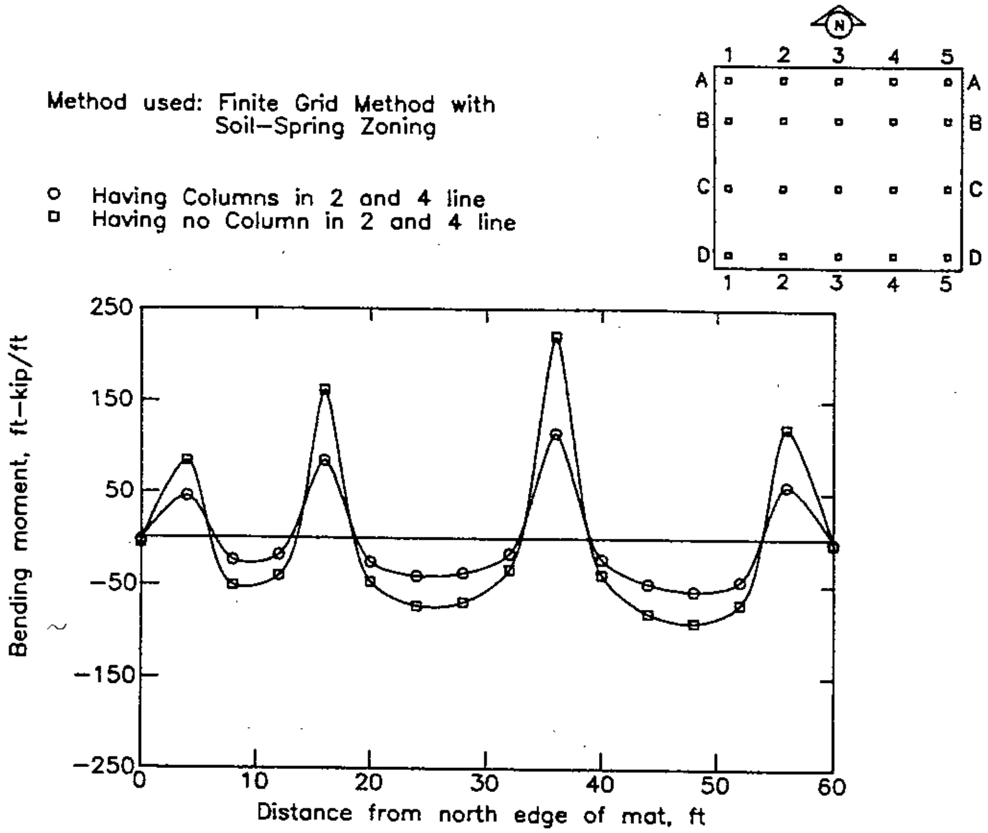


Fig.5.71 Variation of bending moment along column line 3 with and without columns in 2 and 4 line

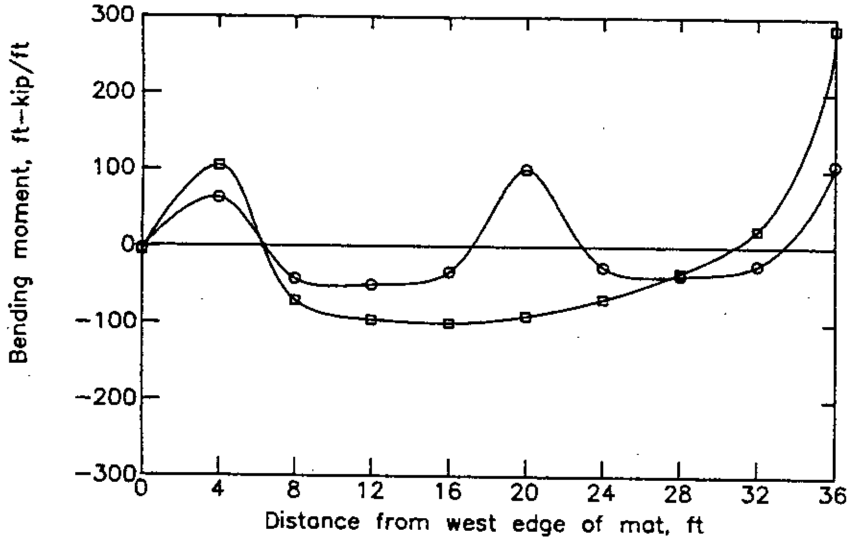


Fig.5.72 Variation of bending moment along column line C with and without columns in 2 and 4 line

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

6.1 CONCLUSIONS

The following are the conclusions that can be drawn from the preceding chapters :

(1) Conventional method of analysis has been found to be inadequate to give realistic results in the analysis of mat foundations. There are two reasons; firstly in this method the mat is analysed strip by strip with uniform soil pressure acting under the strip which causes large moments between columns and small moments at column points. Secondly, the high moments at column points are flattened when the total moment in a strip is divided over the whole width.

(2) ACI approximate flexible method is also inadequate since it is based on functions derived for infinite plates. Although end conditioning forces are applied to simulate a finite mat, they are applied in strips and cause concentrated effects at the edges. But this method yields results more or less comparable to discrete element methods at points far from the edges.

(3) Finite difference method is based on the basic differential equation of plates resting on Winkler's medium and has been proven to yield results comparable to other refined discrete element approaches.

(4) Finite grid method utilises the techniques of finite element. This method employs beam column elements having only bending, torsion and shear at both the edges. It has proven to give results comparable to other discrete element methods.

(5) By comparing the (positive) moments at column points obtained by different methods, it has been found that the maximum moment is due to finite grid method with soil spring zoning. Finite grid method without soil spring zoning and finite difference method yield next higher values. ACI approximate flexible method gives least value among comparable methods.

As for (negative) moments between column points, the maximum moment among comparable methods is due to finite grid method without soil spring zoning. The finite grid method with zoning, finite difference method and ACI method yield next higher moments respectively.

(6) Finite grid method without soil spring zoning and finite difference method yield almost equal deflections at all the points. But with soil spring zoning, the deflections towards the centre of the mat are found to be much higher. ACI method yields deflections which are more or less comparable to discrete element methods at points away from the edge of the mat.

(7) The method used to couple soil springs i.e. zoning of them has been found to simulate the mat behaviour more accurately. The deflected shape of a mat should be like a dish i.e. deflection towards the centre should be higher than the edges. Also the corners of a mat should deflect less due to the effect as in the case of a plate. These behaviours are found when soil spring zoning is employed.

(8) The effect of modulus of subgrade reaction on deflection of mat is much pronounced than its effect on moment. In all the methods, mat deflections decrease almost exponentially with an increase in modulus of soil. At most column points, increase in moment has been found with increase in modulus. But at some edge points, there is a reduction in moment due to more stiffening of interior columns than exterior ones. With soil spring zoning the reverse has been found at some cases. In these cases, although both exterior and interior column points become stiffer than their no zoning counterparts, the larger share of the increase in stiffness is borne by the exterior column points.

(9) Increase in mat thickness increases flexural rigidity and as the mat becomes stiffer, the load transferred from column to soil is more uniformly distributed around column and, thus column moment is reduced. Finite difference and finite grid method without soil spring zoning show this reduction at all the column points. But with spring zoning, at some interior column points, there are increases in moments. Zoning decreases the stiffness of interior columns more compared to exterior columns. When stiffness is added by increasing mat thickness, the ratio of stiffnesses of exterior- and interior-column decreases and consequently, higher moment than before are found at interior columns. All the

methods yield lower deflections at all the points with increase in mat thickness, as it should.

(10) With the increase in Poisson's ratio, moments in both the directions increase in all the methods. However, this increase is more pronounced in finite difference method and almost negligible in case the of finite grid method. Poisson's ratio has almost no effect on deflection of mat irrespective of the method used.

(11) Flexural rigidity of mat increases with an increase in concrete strength. A stiffer mat transfers load to soil more uniformly and column moments are thereby reduced. All the discrete element methods have shown this reduction in moment in columns although the magnitude of such reduction are very small. At some interior points, finite grid method with soil spring zoning has shown slightly higher moment for higher f'_c due to the relative increase in stiffness at interior points.

(12) It has been found that if column loads are doubled, keeping mat plan dimensions and other properties unchanged, the moment and deflections at all the points are also doubled. It is, of course, obvious that keeping stiffness of the mat-soil system unchanged, change in external loads results in a corresponding change in deflections and moments accordingly, as it should in a linear analysis.

(13) With the increase in column loads on one side of the centroid of mat, as in the case of wind load, little change has been observed in moments on the other side. But there is substantial increase in moments and deflections on the side where loads are increased. A slight uplift on the other side of increased load has been observed.

(14) It has been found that if spans remain unchanged in a direction of mat, the summation of column moments in that direction remains unchanged no matter how many parallel column lines are present. Total column moment in a direction is distributed among parallel column lines according to their contributing areas.

6.2 ASSESSMENT OF DIFFERENT METHODS

6.2.1 Conventional Method

Merits

(1) The method is simple and hand computation is sufficient.

(2) Solution of the whole mat is not needed; rather a strip of interest can be solved alone.

Demerit

(1) The method is inadequate to give satisfactory results.

6.2.2 ACI Approximate Flexible Method

Merits

(1) The program is simple in nature and input data is easy to produce. The output can also be interpreted easily.

(2) The computer time and memory required are minimum.

(3) The program can be used as a spot check for a certain point on mat without solving the whole mat.

Demerits

(1) The method is approximate and based on infinite mat resting on soil springs and is not mathematically elegant.

(2) Column moments can not be included.

(3) Boundary conditions can not be modelled.

(4) Application of end conditioning forces to represent a mat of finite size results in unsatisfactory deflections and moments near edges. But at points away from the edges, calculated moments and deflections are comparable to discrete element methods.

6.2.3 Finite Difference Method

Merits

(1) The input data required are small in number and easier to produce.

(2) The output is also simple to interpret.

(3) The method uses basic differential equation of plates resting on soil springs and thus it is mathematically elegant compared to approximate methods.

Demerits

(1) The solution procedure used in the present program is slow and the computer time required is very high compared to other methods under consideration.

(2) It requires a large computer memory as huge temporary files are created to store data during the solving procedure.

(3) Although the grid may be rectangular, it requires a constant grid spacing in each direction. This requirement makes it difficult to model mats of arbitrary configuration.

(4) It is not possible to model boundary conditions such as points of zero rotation, column fixity etc.

(5) Column moments can not be modelled.

(6) No method for soil spring coupling can be used.

6.2.4 Finite Grid Method

Merits

(1) This method uses the techniques of finite element and is mathematically sound.

(2) Formulation and solution of stiffness matrix in the present program are quite rapid.

(3) Constant grid spacing is not required and thus any mat configuration can be modelled.

(4) Different soil spring values at different points can be used which enables the soil spring zoning method to be employed to couple the soil springs.

(5) Boundary conditions can be modelled.

(6) Column moments can be included.

Demerit

(1) The program requires large input data, but this problem has been solved employing novel data generation program.

6.3 RECOMMENDATIONS

The following recommendations are made for future development of the present research work :

(1) The present computer program of finite grid method uses beam element with torsion at two ends. Finite element program can be developed using plate elements for better representation of a mat.
(2) Soil is represented by discrete springs in all the methods of the study. Solid soil elements can be employed in a finite element program for simulation of a mat-soil system.
(3) The possible development of crack in soil is considered in finite grid method but the same within mat concrete is not considered. This can be taken into consideration in future research.
(4) The solution procedure of finite difference program can be improved to make it more efficient.
(5) There is actually very little practical data available concerning the behaviour of mat foundations; a physical model may be constructed and tested to get a better insight into the actual response of a mat-soil system under load.

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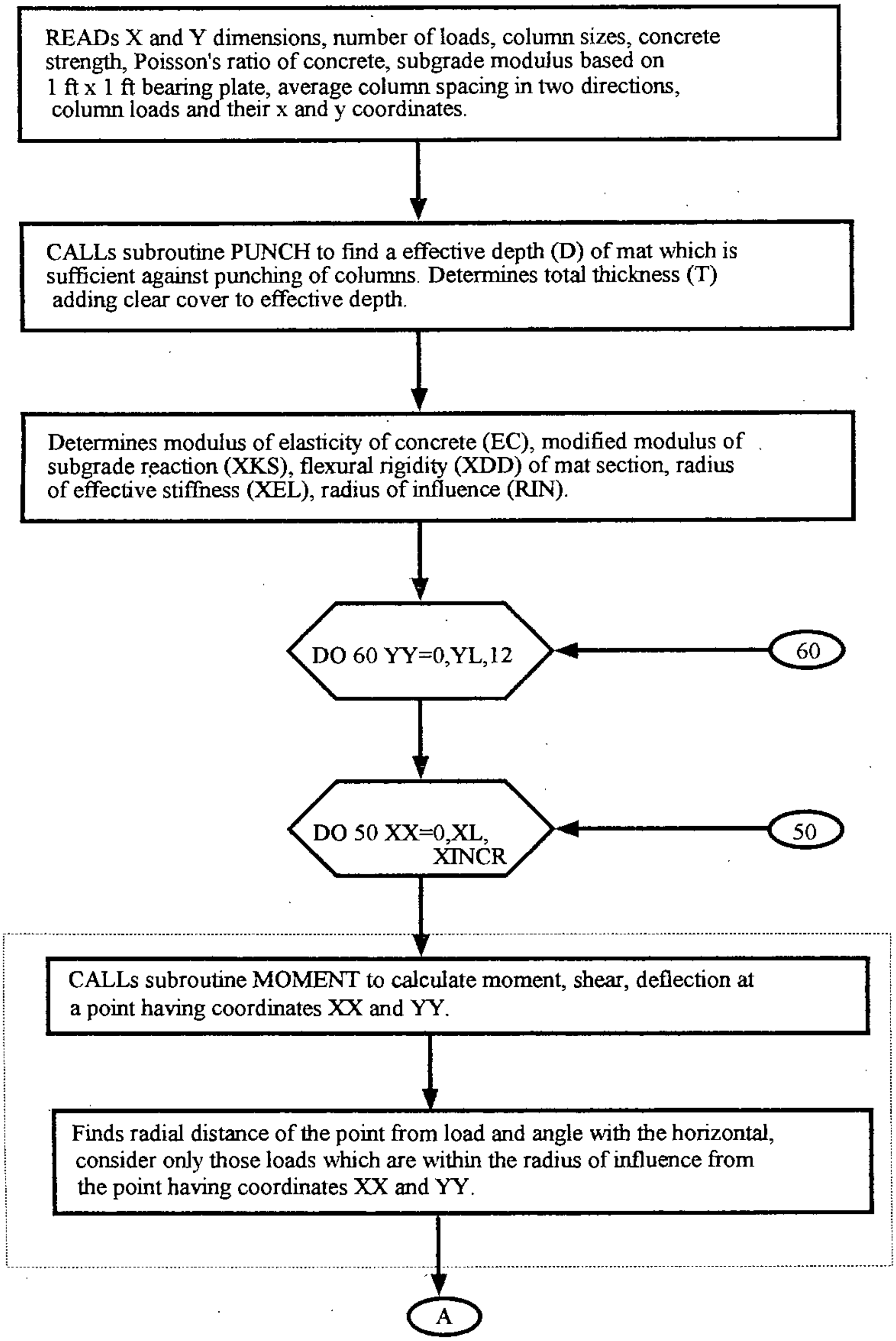
APPENDIX-A

FLOW CHART OF PROGRAM OF ACI APPROXIMATE FLEXIBLE METHOD



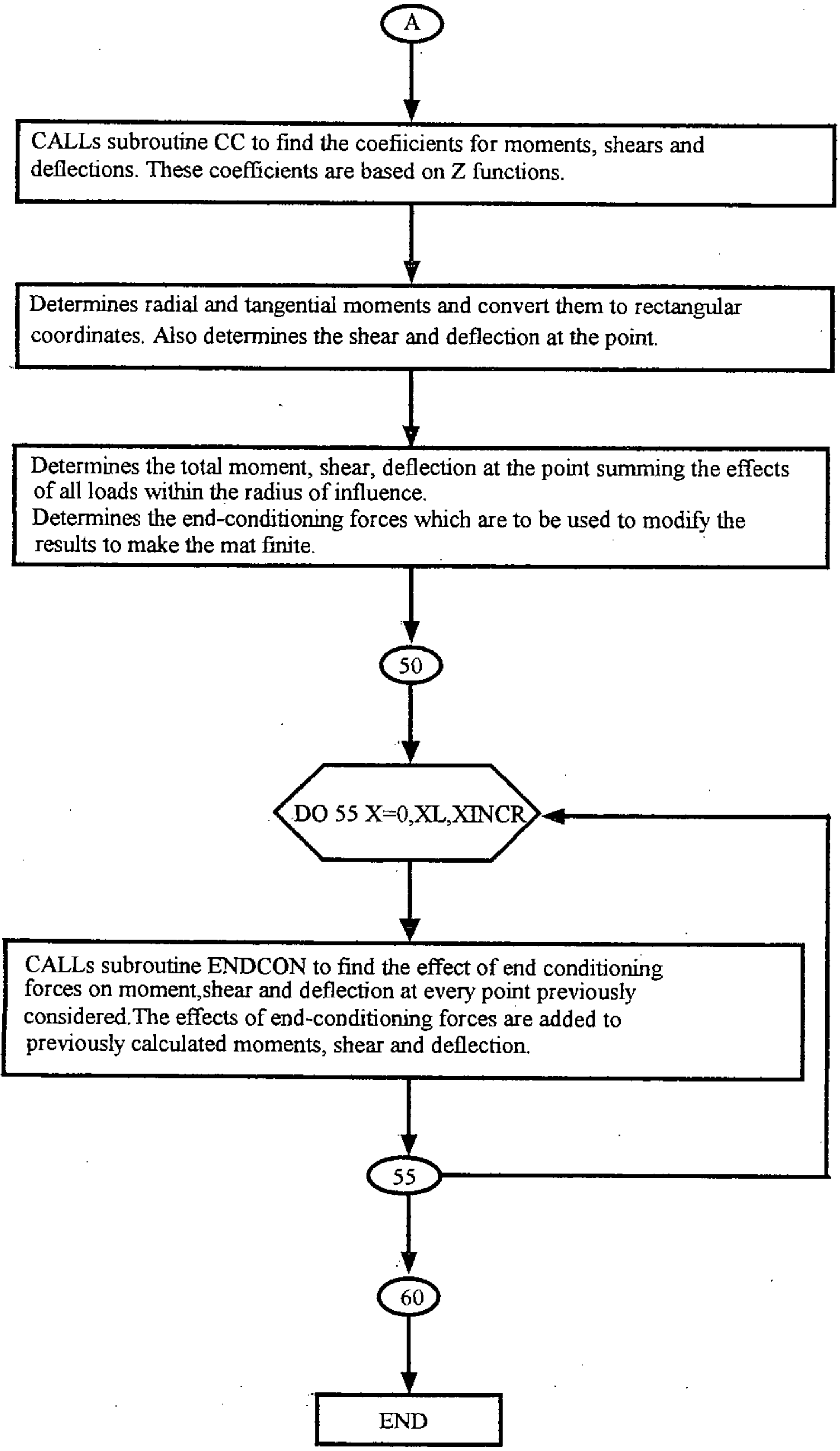
READs X and Y dimensions, number of loads, column sizes, concrete strength, Poisson's ratio of concrete, subgrade modulus based on 1 ft x 1 ft bearing plate, average column spacing in two directions, column loads and their x and y coordinates.

CALLs subroutine PUNCH to find a effective depth (D) of mat which is sufficient against punching of columns. Determines total thickness (T) adding clear cover to effective depth.

Determines modulus of elasticity of concrete (EC), modified modulus of subgrade reaction (XKS), flexural rigidity (XDD) of mat section, radius of effective stiffness (XEL), radius of influence (RIN).

CALLs subroutine MOMENT to calculate moment, shear, deflection at a point having coordinates XX and YY.

Finds radial distance of the point from load and angle with the horizontal, consider only those loads which are within the radius of influence from the point having coordinates XX and YY.



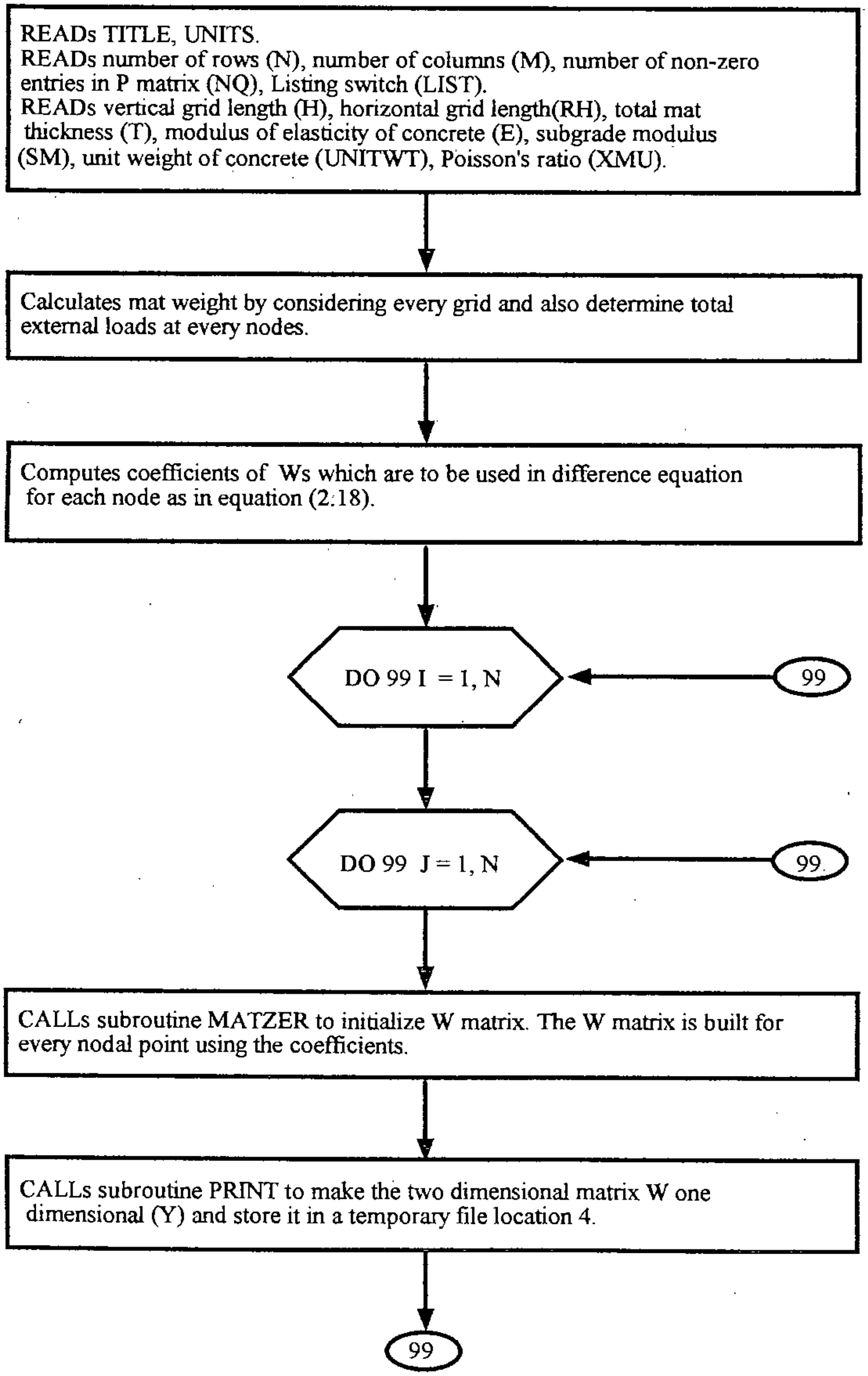
CALLs subroutine CC to find the coefiicients for moments, shears and deflections. These coefficients are based on Z functions.

Determines radial and tangential moments and convert them to rectangular coordinates. Also determines the shear and deflection at the point.

Determines the total moment, shear, deflection at the point summing the effects of all loads within the radius of influence.
Determines the end-conditioning forces which are to be used to modify the results to make the mat finite.

CALLs subroutine ENDCON to find the effect of end conditioning forces on moment,shear and deflection at every point previously considered.The effects of end-conditioning forces are added to previously calculated moments, shear and deflection.

FLOW CHART OF THE PROGRAM OF FINITE DIFFERENCE METHOD



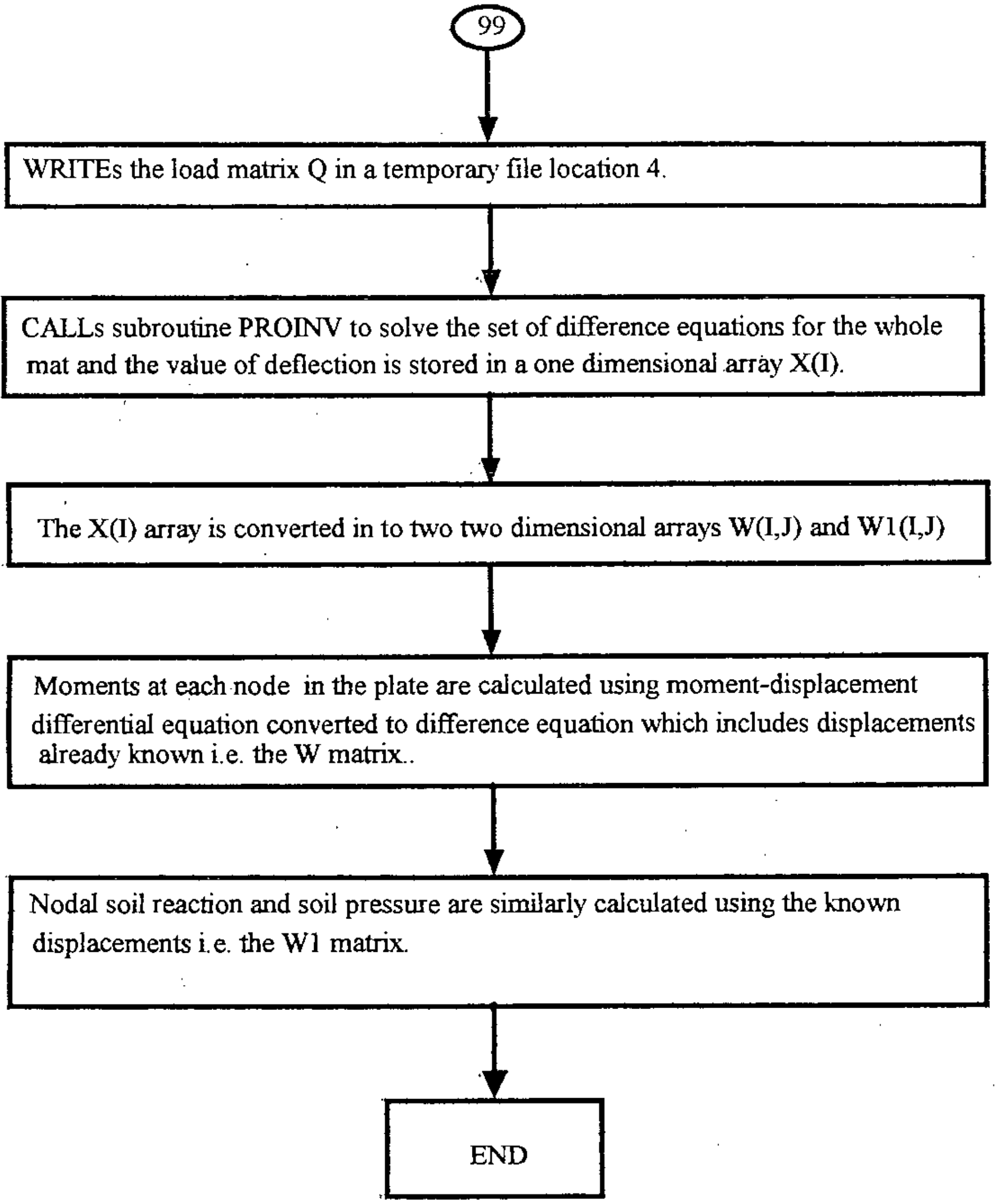
READs TITLE, UNITS.
READs number of rows (N), number of columns (M), number of non-zero entries in P matrix (NQ), Listing switch (LIST).
READs vertical grid length (H), horizontal grid length(RH), total mat thickness (T), modulus of elasticity of concrete (E), subgrade modulus (SM), unit weight of concrete (UNITWT), Poisson's ratio (XMU).

Calculates mat weight by considering every grid and also determine total external loads at every nodes.

Computes coefficients of Ws which are to be used in difference equation for each node as in equation (2.18).

CALLs subroutine MATZER to initialize W matrix. The W matrix is built for every nodal point using the coefficients.

CALLs subroutine PRINT to make the two dimensional matrix W one dimensional (Y) and store it in a temporary file location 4.



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WRITEs the load matrix Q in a temporary file location 4.

CALLs subroutine PROINV to solve the set of difference equations for the whole mat and the value of deflection is stored in a one dimensional array X(I).

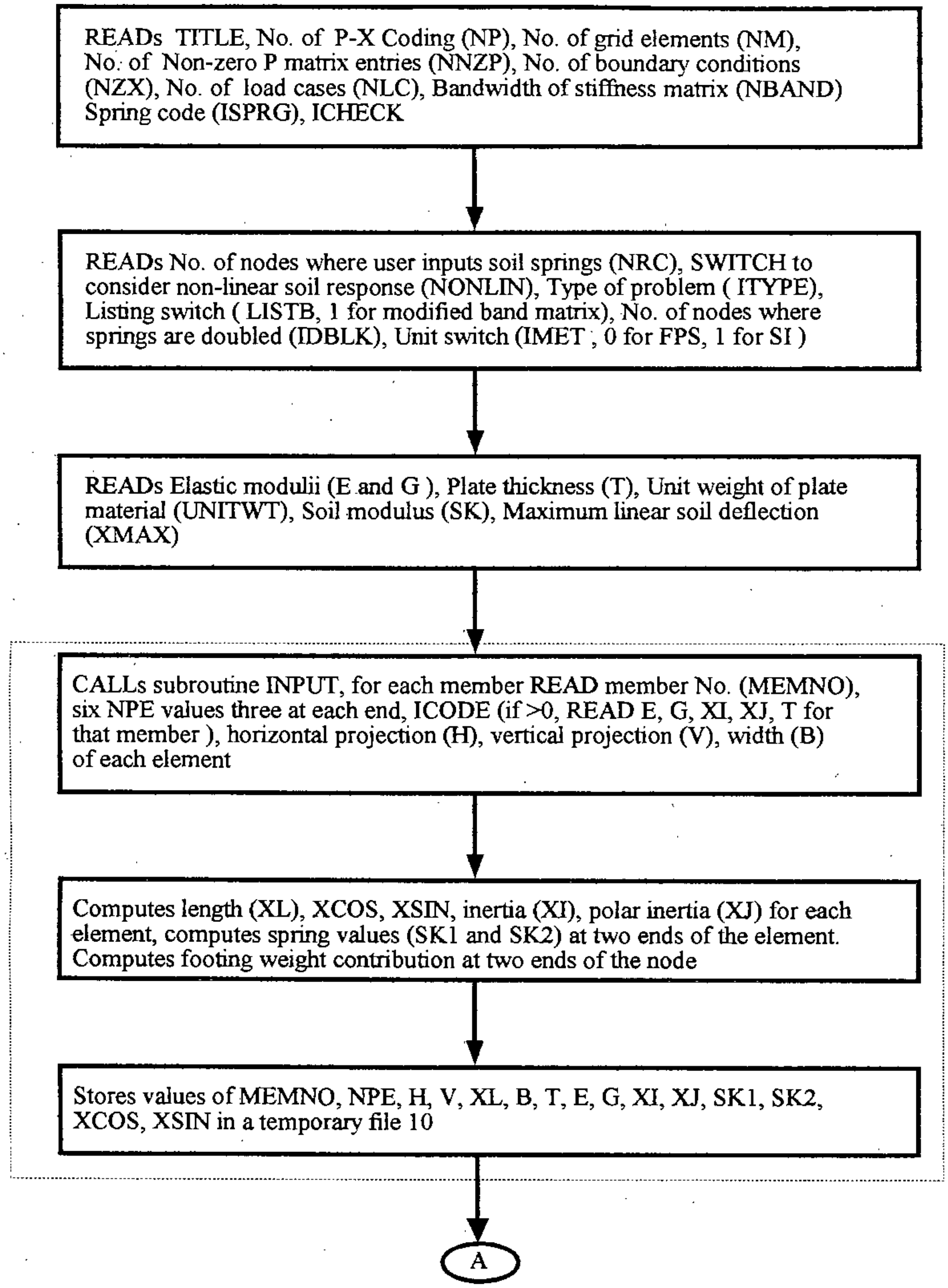
The X(I) array is converted in to two two dimensional arrays W(I,J) and W1(I,J)

Moments at each node in the plate are calculated using moment-displacement differential equation converted to difference equation which includes displacements already known i.e. the W matrix..

Nodal soil reaction and soil pressure are similarly calculated using the known displacements i.e. the W1 matrix.

END

FLOW CHART OF PROGRAM OF FINITE GRID METHOD



READs TITLE, No. of P-X Coding (NP), No. of grid elements (NM), No. of Non-zero P matrix entries (NNZP), No. of boundary conditions (NZX), No. of load cases (NLC), Bandwidth of stiffness matrix (NBAND) Spring code (ISPRG), ICHECK

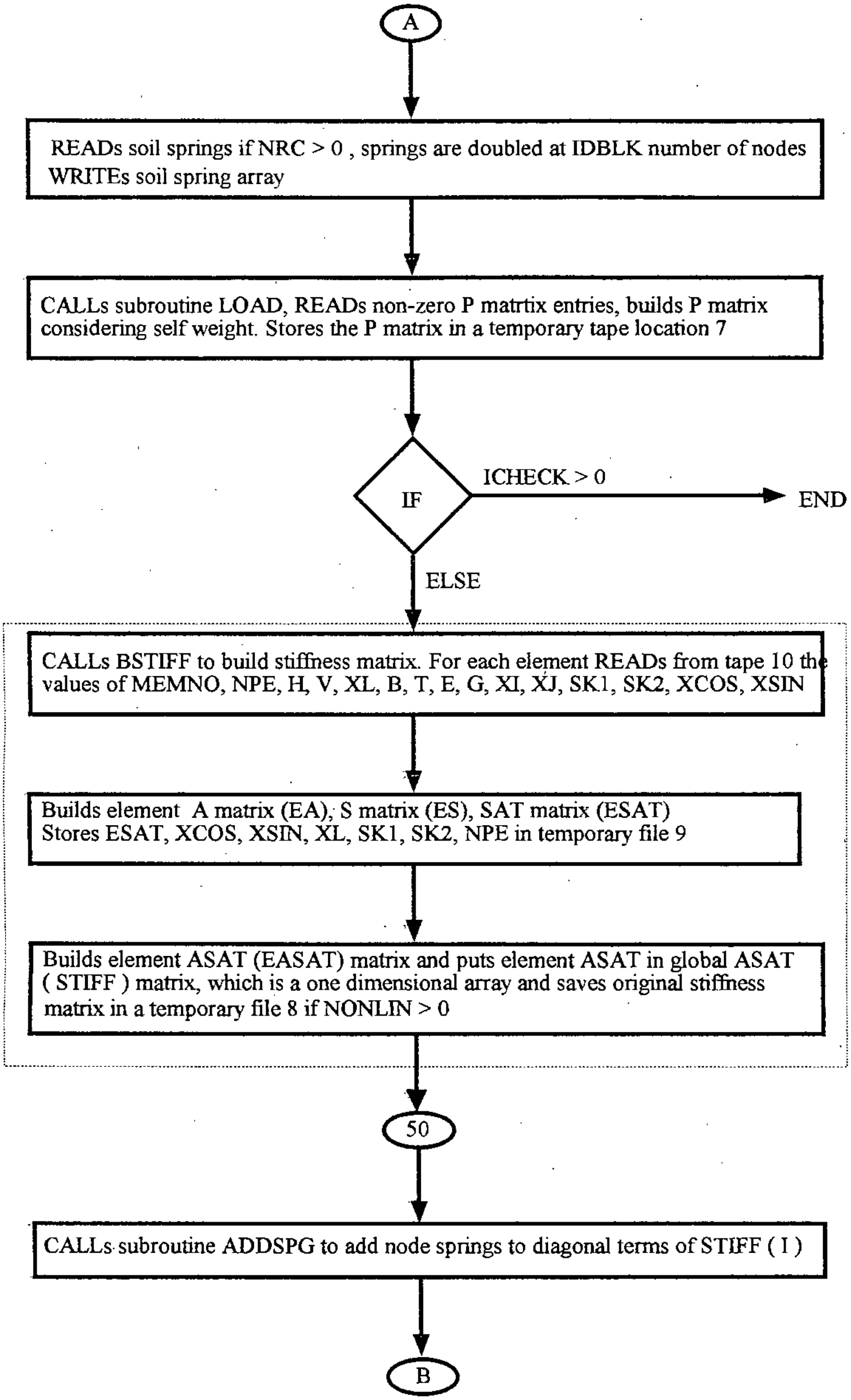
READs No. of nodes where user inputs soil springs (NRC), SWITCH to consider non-linear soil response (NONLIN), Type of problem (ITYPE), Listing switch (LISTB, 1 for modified band matrix), No. of nodes where springs are doubled (IDBLK), Unit switch (IMET ; 0 for FPS, 1 for SI)

READs Elastic modulii (E and G), Plate thickness (T), Unit weight of plate material (UNITWT), Soil modulus (SK), Maximum linear soil deflection (XMAX)

CALLs subroutine INPUT, for each member READ member No. (MEMNO), six NPE values three at each end, ICODE (if >0, READ E, G, XI, XJ, T for that member), horizontal projection (H), vertical projection (V), width (B) of each element

Computes length (XL), XCOS, XSIN, inertia (XI), polar inertia (XJ) for each element, computes spring values (SK1 and SK2) at two ends of the element. Computes footing weight contribution at two ends of the node

Stores values of MEMNO, NPE, H, V, XL, B, T, E, G, XI, XJ, SK1, SK2, XCOS, XSIN in a temporary file 10



READs soil springs if NRC > 0 , springs are doubled at IDBLK number of nodes
WRITEs soil spring array

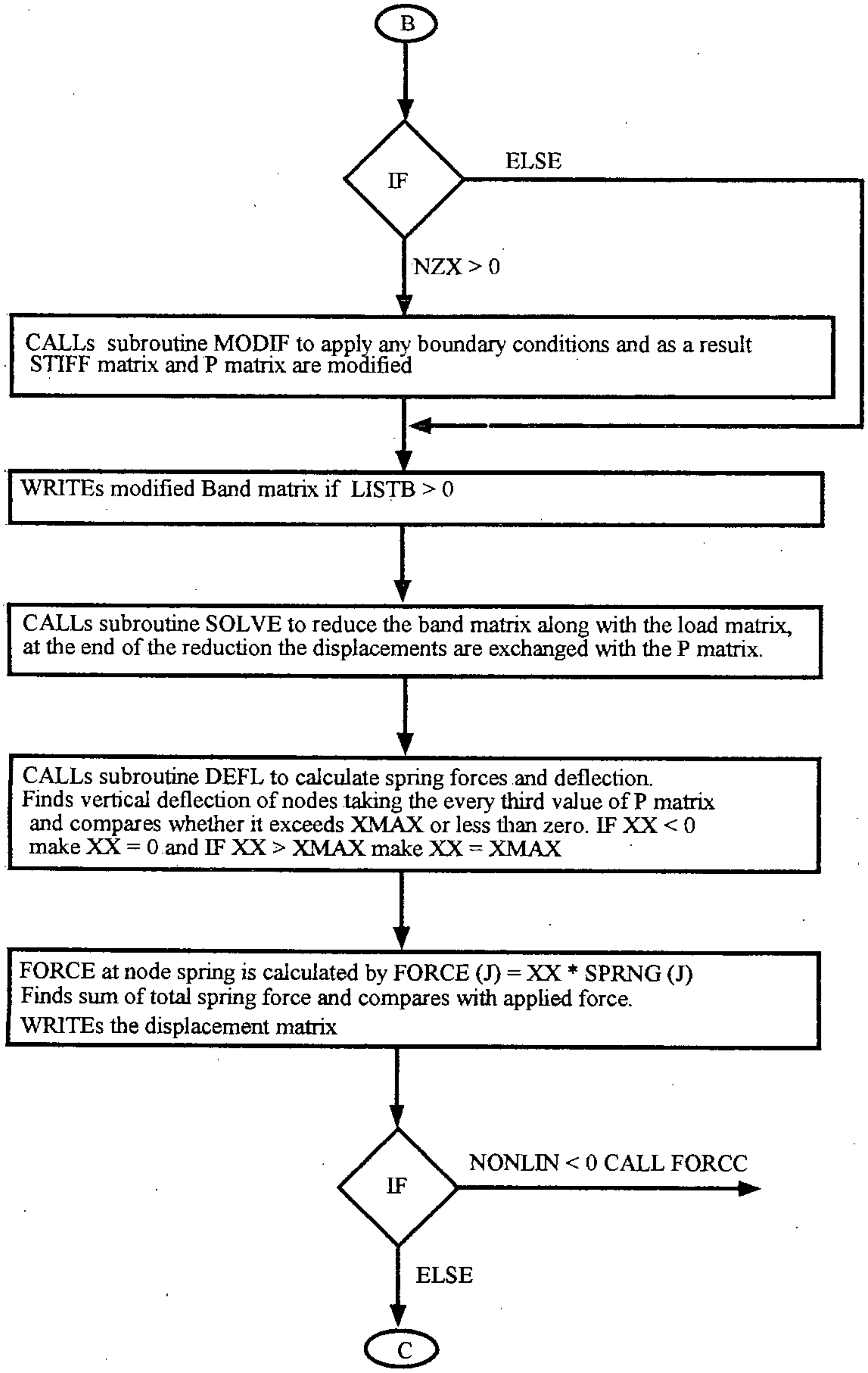
CALLs subroutine LOAD, READs non-zero P matrtix entries, builds P matrix considering self weight. Stores the P matrix in a temporary tape location 7

CALLs BSTIFF to build stiffness matrix. For each element READs from tape 10 th values of MEMNO, NPE, H, V, XL, B, T, E, G, XI, XJ, SK1, SK2, XCOS, XSIN

Builds element A matrix (EA), S matrix (ES), SAT matrix (ESAT)
Stores ESAT, XCOS, XSIN, XL, SK1, SK2, NPE in temporary file 9

Builds element ASAT (EASAT) matrix and puts element ASAT in global ASAT (STIFF) matrix, which is a one dimensional array and saves original stiffness matrix in a temporary file 8 if NONLIN > 0

CALLs subroutine ADDSPG to add node springs to diagonal terms of STIFF (I)



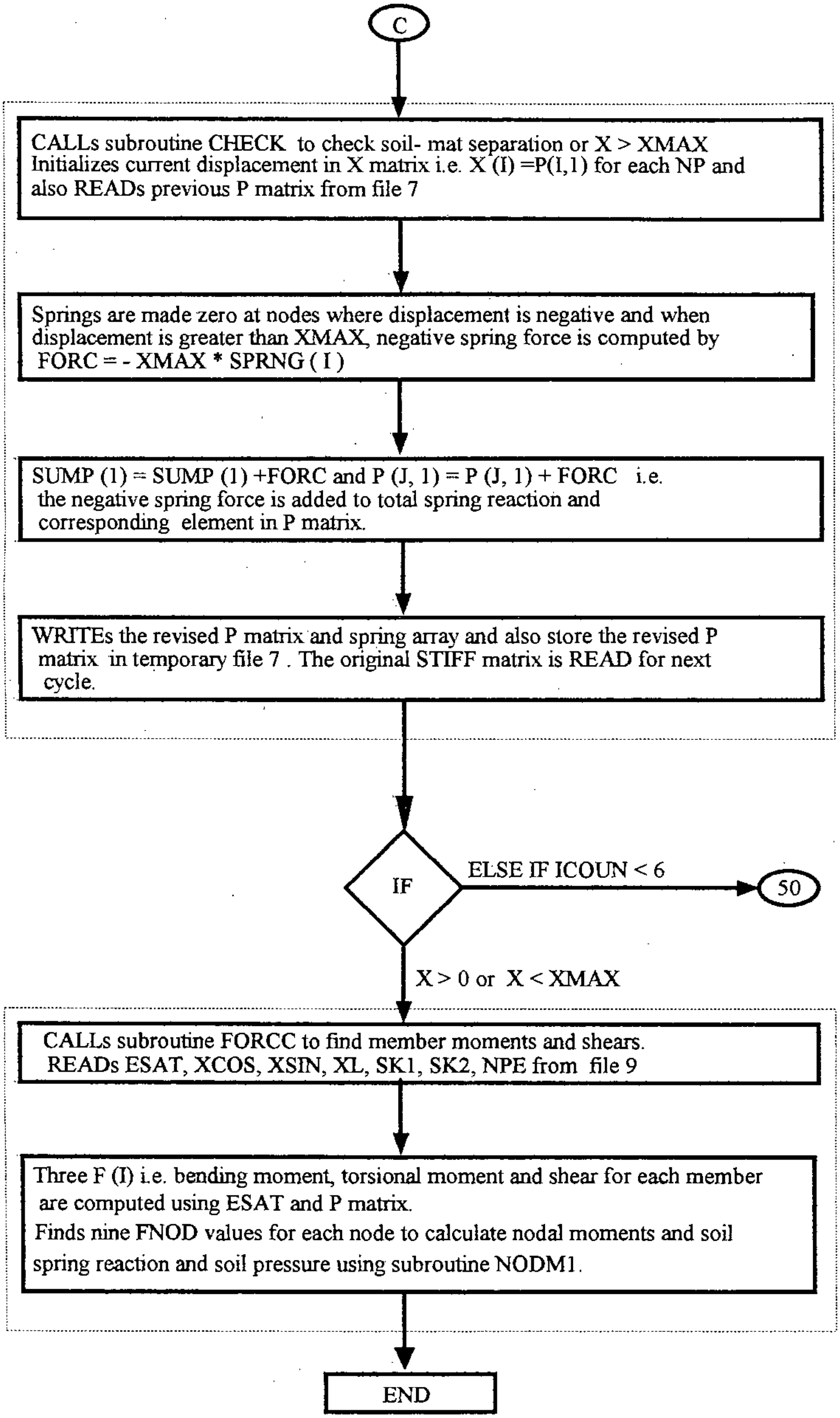
CALLs subroutine MODIF to apply any boundary conditions and as a result STIFF matrix and P matrix are modified

WRITEs modified Band matrix if LISTB > 0

CALLs subroutine SOLVE to reduce the band matrix along with the load matrix, at the end of the reduction the displacements are exchanged with the P matrix.

CALLs subroutine DEFL to calculate spring forces and deflection. Finds vertical deflection of nodes taking the every third value of P matrix and compares whether it exceeds XMAX or less than zero. IF XX < 0 make XX = 0 and IF XX > XMAX make XX = XMAX

FORCE at node spring is calculated by FORCE (J) = XX * SPRNG (J)
Finds sum of total spring force and compares with applied force.
WRITEs the displacement matrix



CALLs subroutine CHECK to check soil- mat separation or X > XMAX
Initializes current displacement in X matrix i.e. X (I) =P(I,1) for each NP and also READs previous P matrix from file 7

Springs are made zero at nodes where displacement is negative and when displacement is greater than XMAX, negative spring force is computed by FORC = - XMAX * SPRNG (I)

SUMP (1) = SUMP (1) +FORC and P (J, 1) = P (J, 1) + FORC i.e. the negative spring force is added to total spring reaction and corresponding element in P matrix.

WRITEs the revised P matrix and spring array and also store the revised P matrix in temporary file 7 . The original STIFF matrix is READ for next cycle.

CALLs subroutine FORCC to find member moments and shears.
READs ESAT, XCOS, XSIN, XL, SK1, SK2, NPE from file 9

Three F (I) i.e. bending moment, torsional moment and shear for each member are computed using ESAT and P matrix.
Finds nine FNOD values for each node to calculate nodal moments and soil spring reaction and soil pressure using subroutine NODM1.

APPENDIX-B

Validity of the Program for ACI Approximate Flexible Method

To check whether the program for ACI Approximate Method is correctly coded, three trial cases were run for the mat problem described in article 4.2 with column loads shown in Fig.4.1. To avoid the effect of end-conditioning forces, the mat was made thin and the soil very stiff. Thus the mat could be idealised as infinite with smaller radii of influence not extending beyond the boundaries. It has been found that the total soil pressure agrees with the total column loads within 10 percent variation. Thus the program can be considered as correctly coded as it satisfies vertical forces equilibrium. The results of these three trial cases are presented below:

Thickness of Plate inch	Soil Modulus kcf	ΣColumn Loads kips	ΣSoil Pressure kips	$\frac{\sum \text{Soil Pressure}}{\sum \text{Column Loads}}$
6	5,000	8451.9	9239.2	1.09
6	8,000	8451.9	9107.9	1.08
6	15,000	8451.9	8800.4	1.04

Table B.1 Results showing validity of the program for ACI Approximate Flexible Method

APPENDIX-C

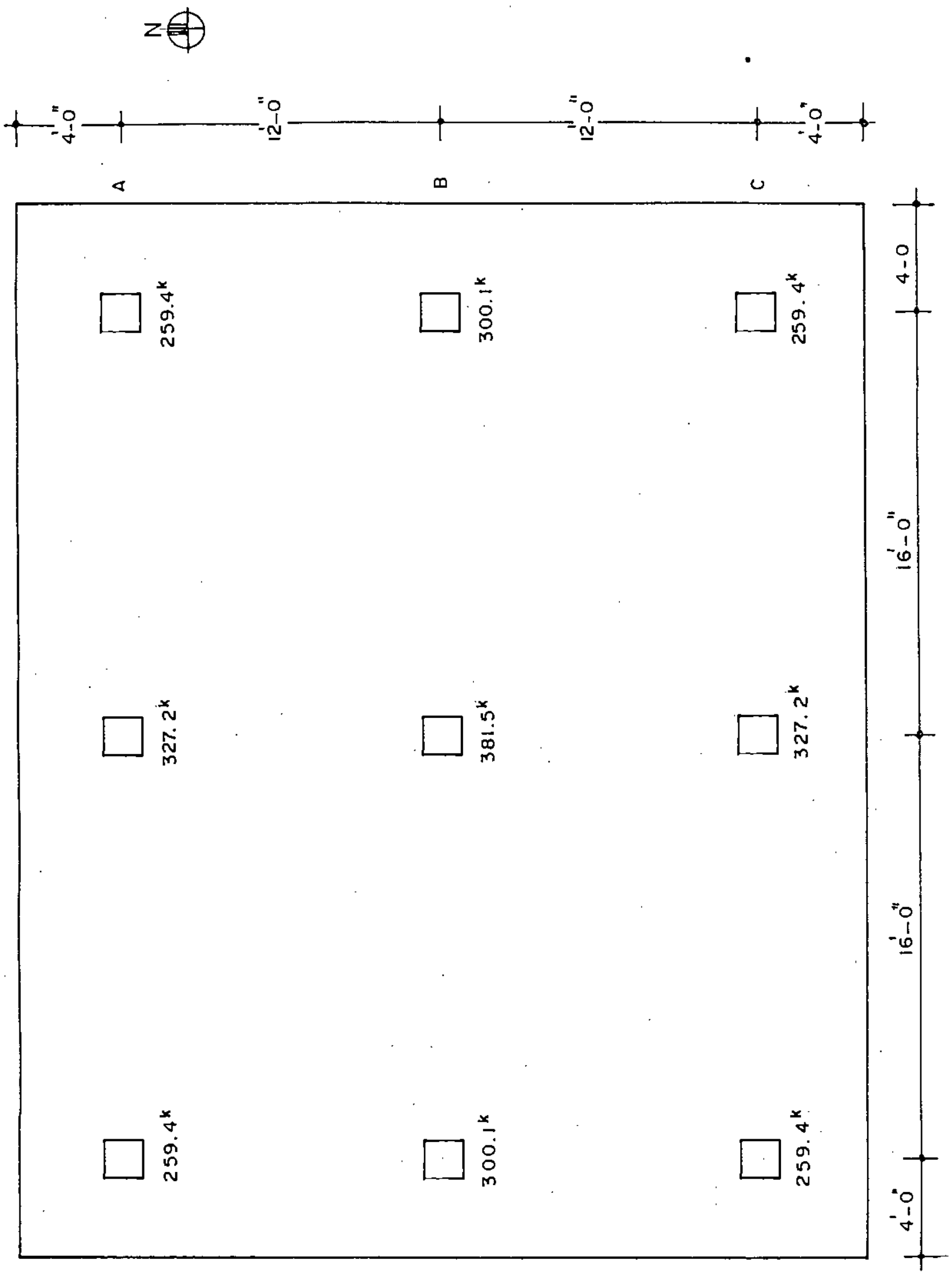


Fig. C.1 Mat for Finite Difference Grid Selection

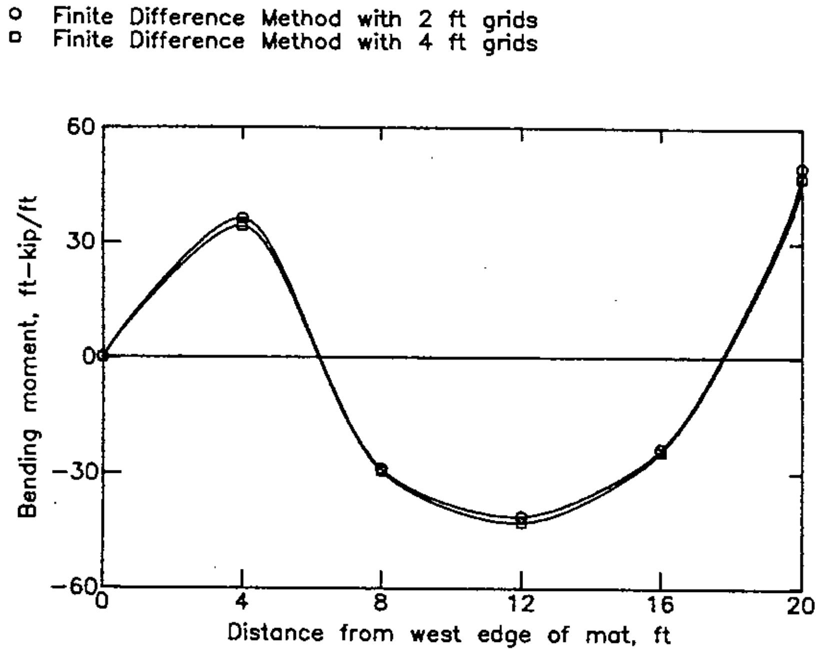


Fig.C.2 Variation of moments along column line B with 2 ft and 4 ft grids

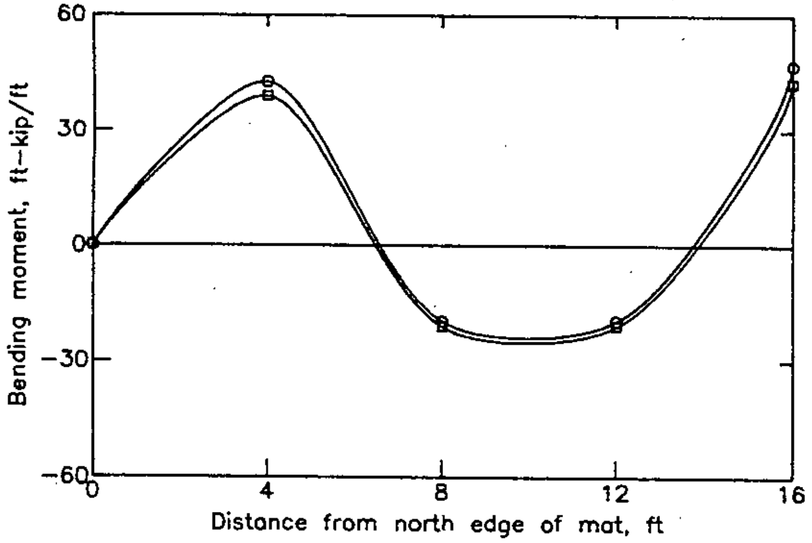


Fig.C.3 Variation of moments along column line 2 with 2 ft and 4 ft grids

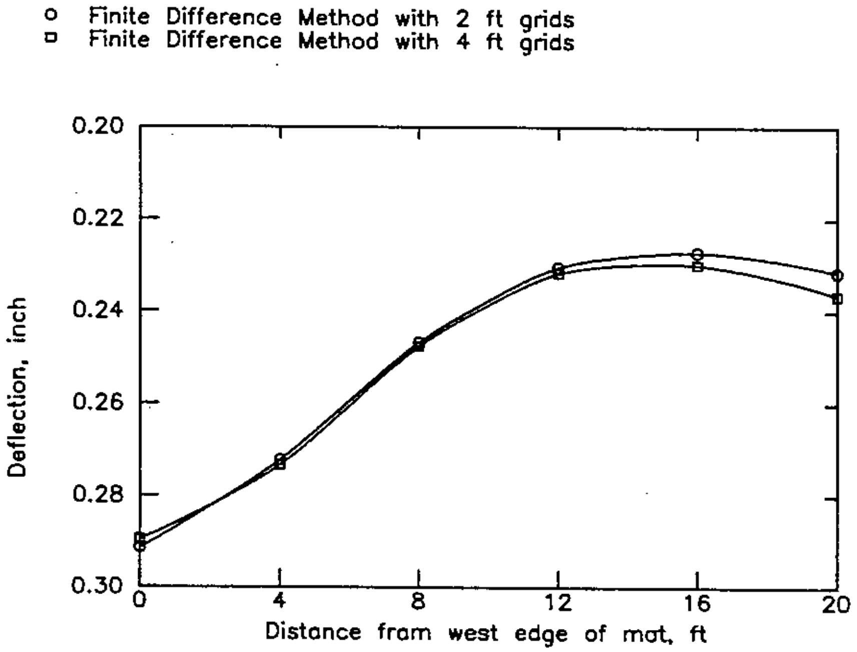


Fig.C.4 Variation of deflections along column line B with 2 ft and 4 ft grids

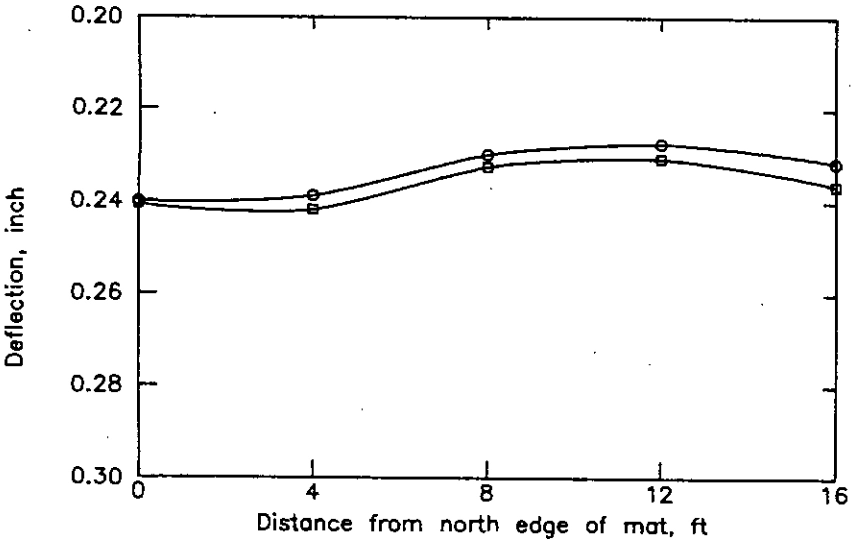


Fig.C.5 Variation of deflections along column line 2 with 2 ft and 4 ft grids

APPENDIX-D

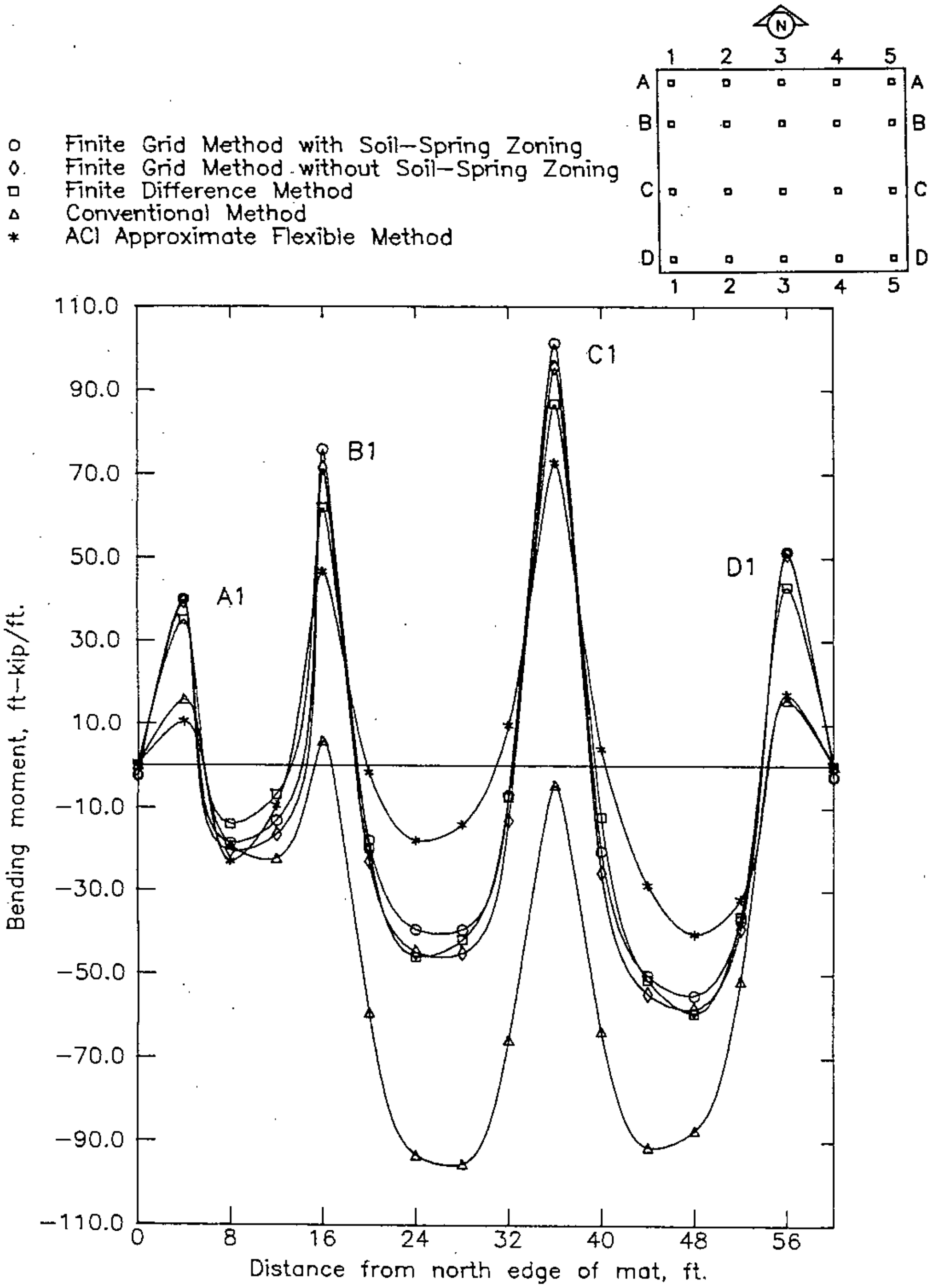


Fig.D.1 Comparison of moments along column line 1

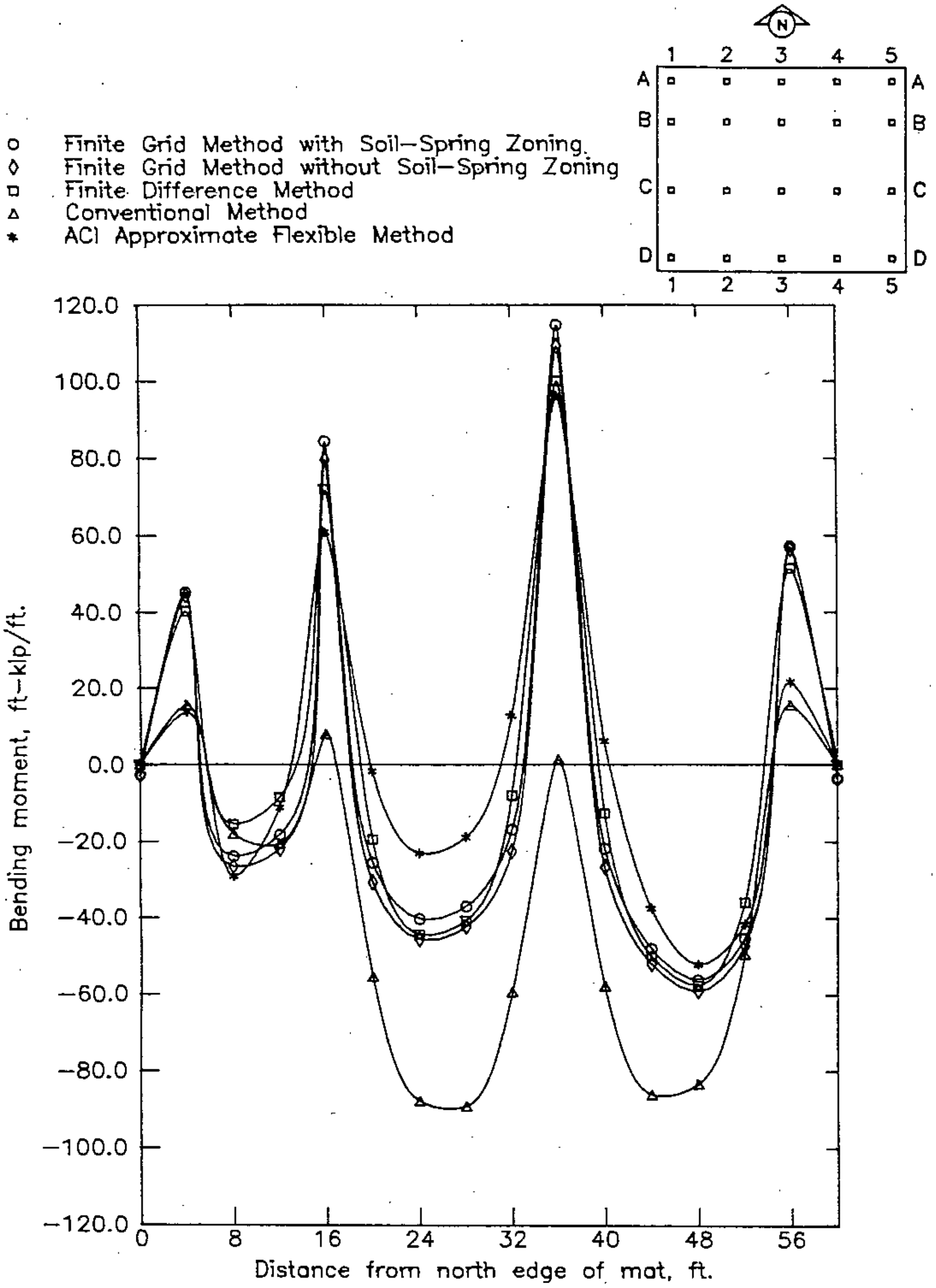


Fig.D.2 Comparison of moments along column line 2

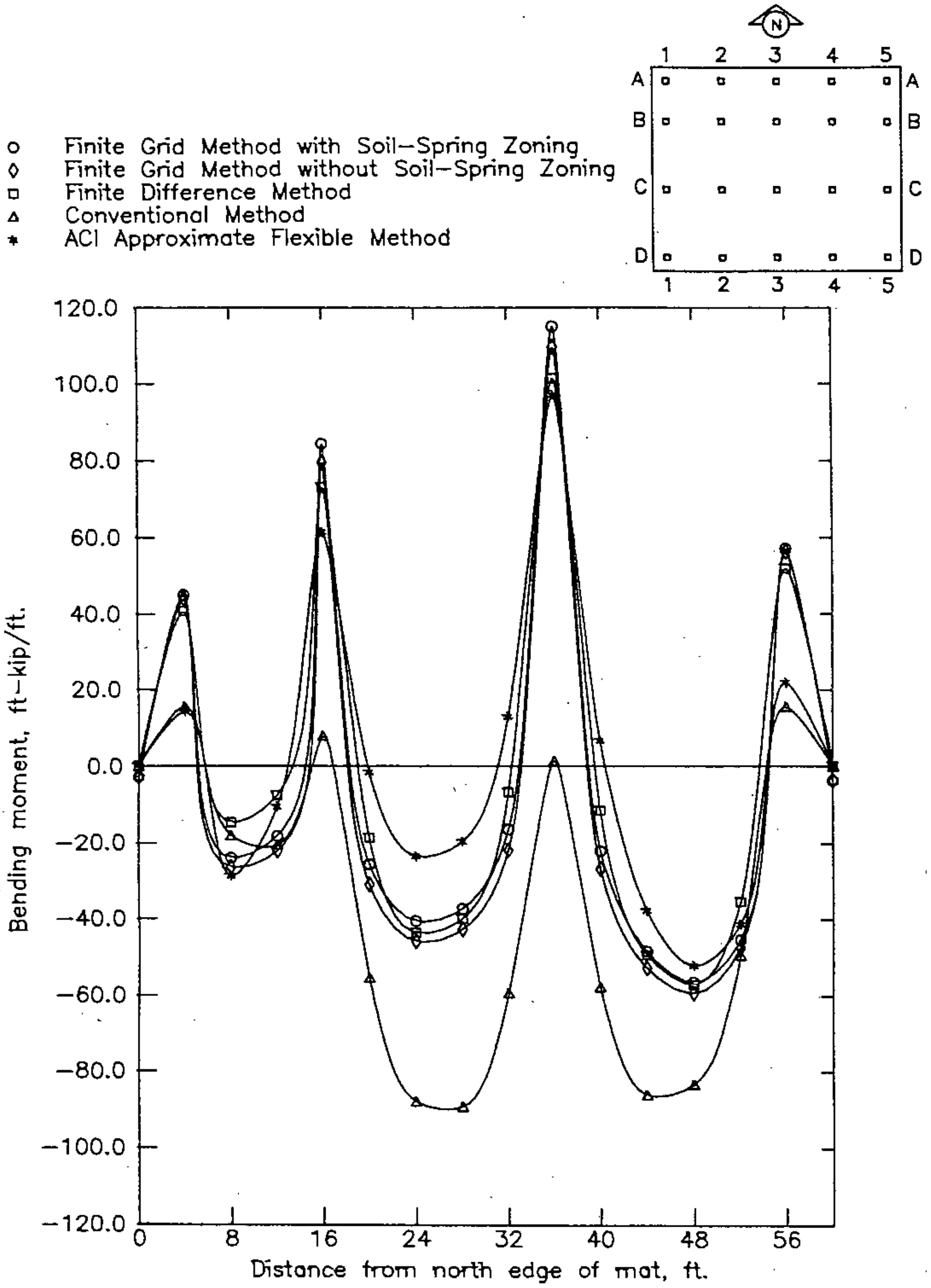


Fig.D.3 Comparison of moments along column line 3

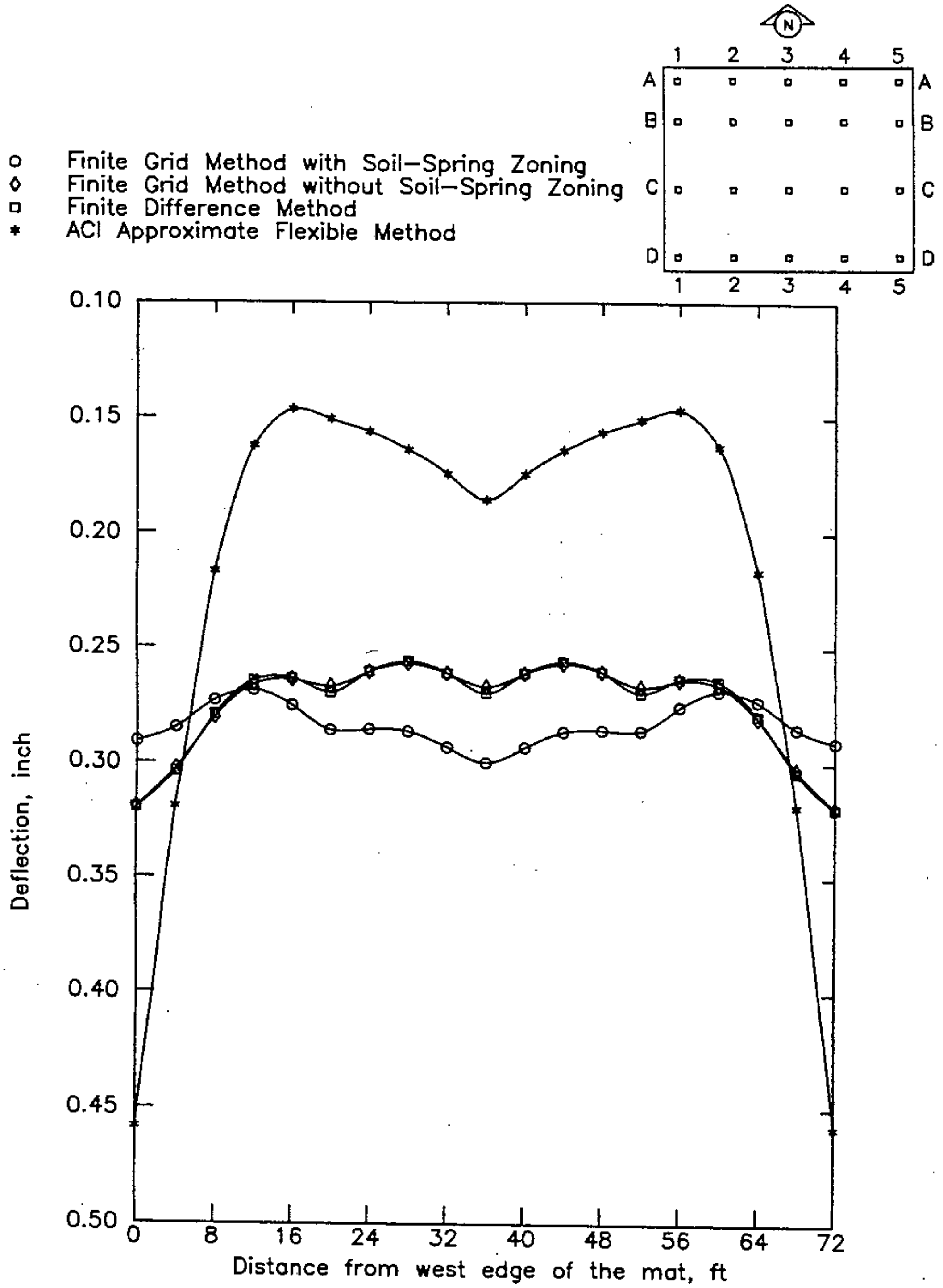


Fig.D.4 Comparison of deflections along column line A

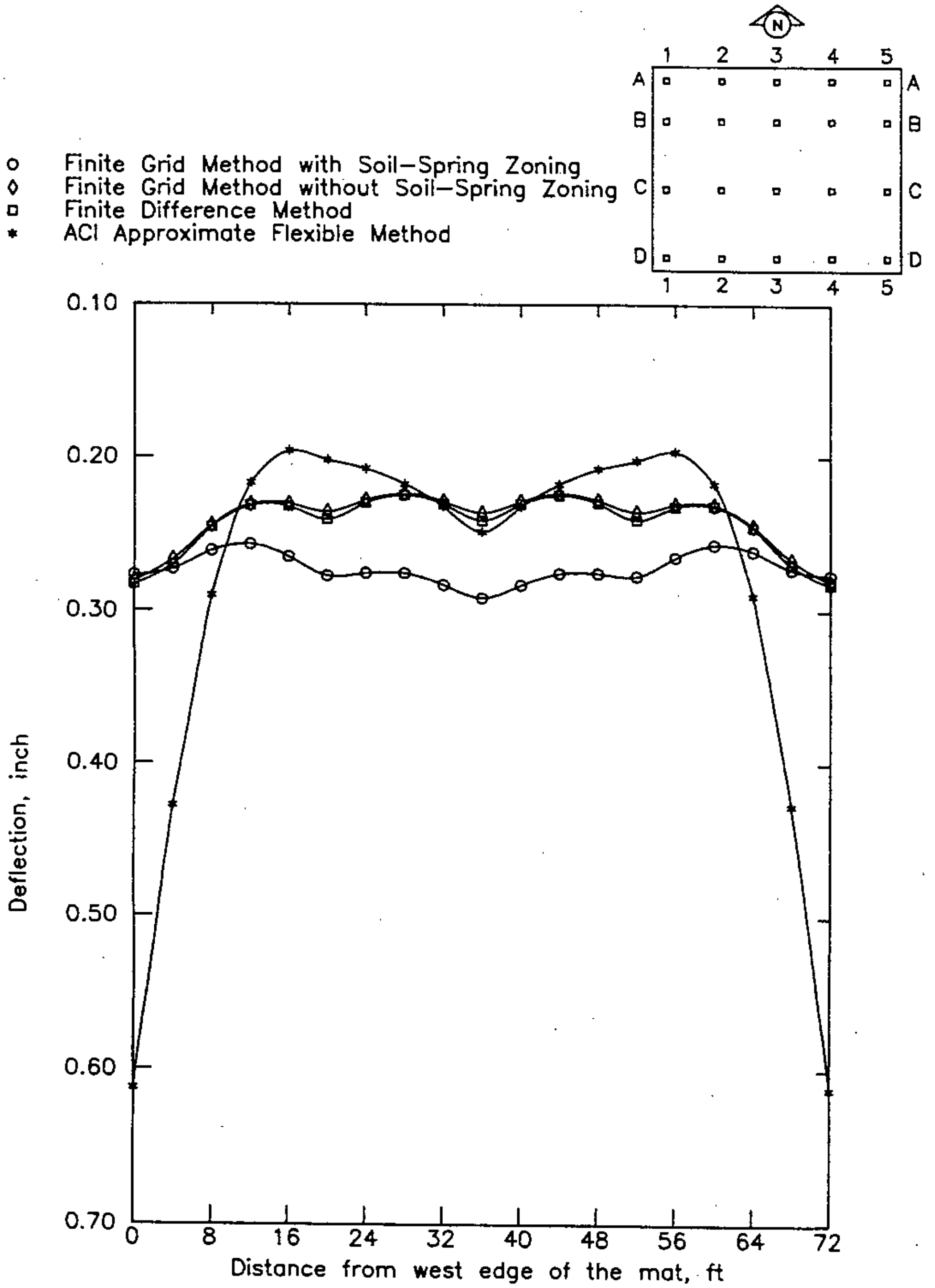


Fig.D.5 Comparison of deflections along column line B

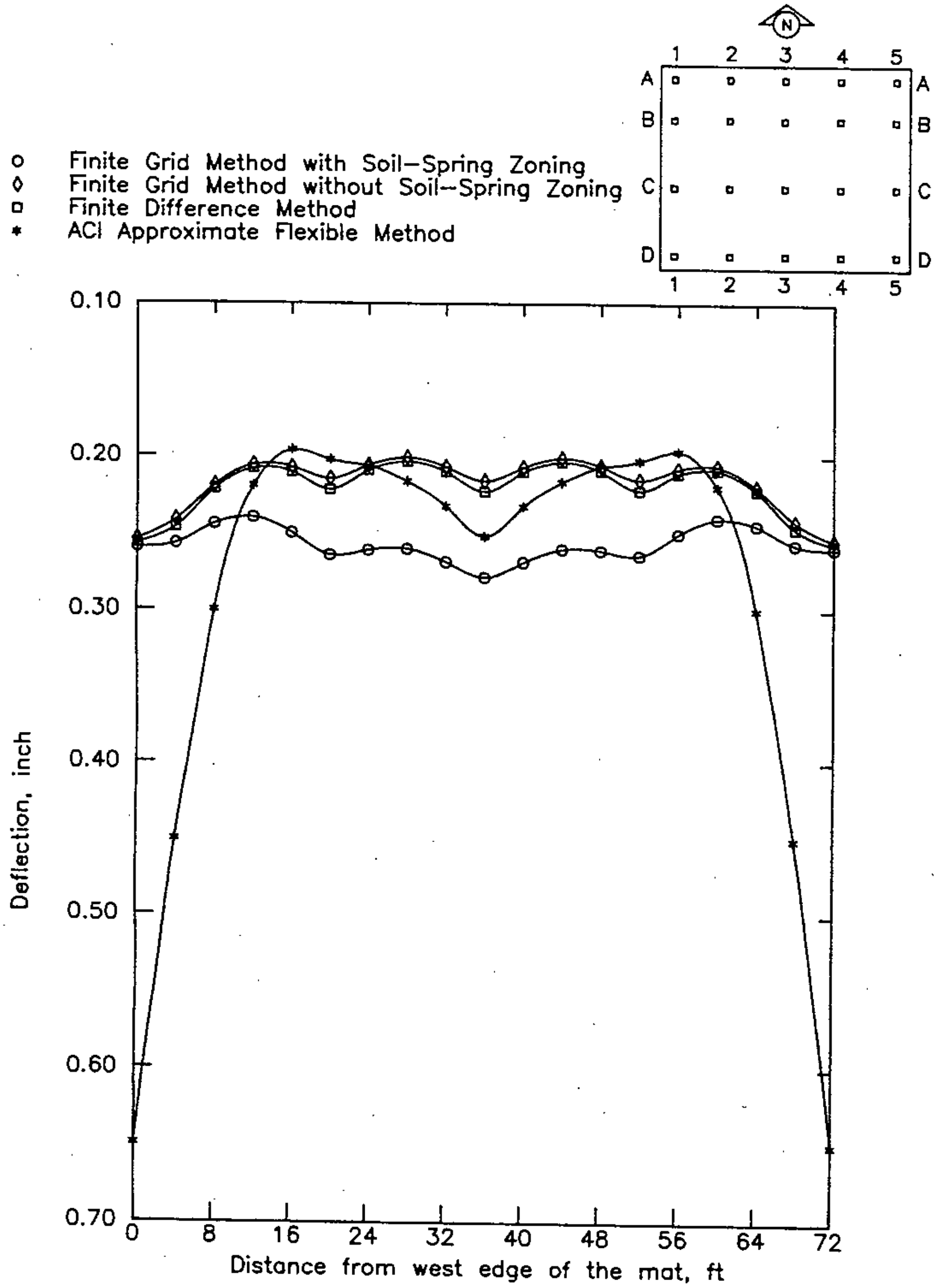


Fig.D.6 Comparison of deflections along column line C

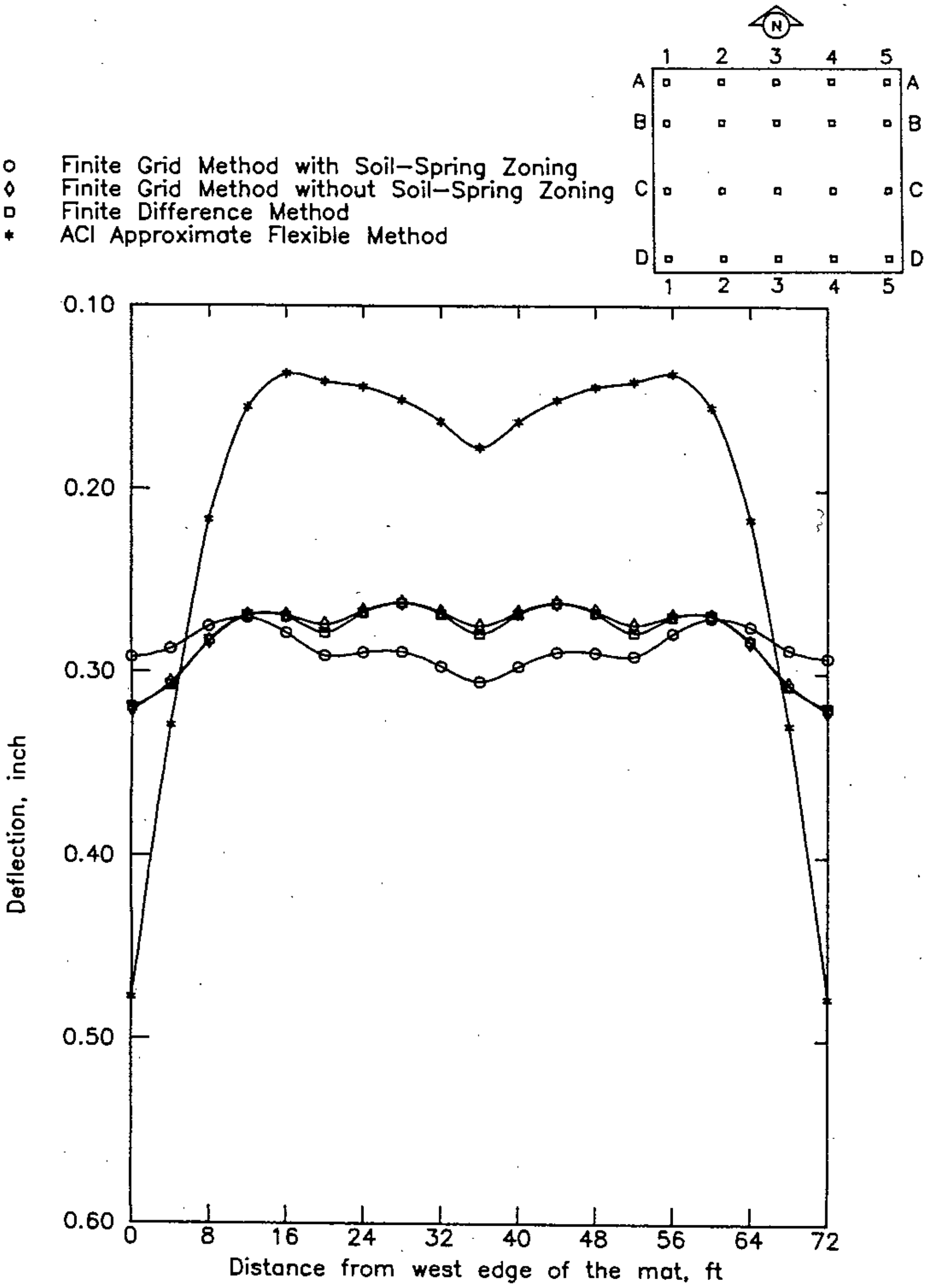


Fig.D.7 Comparison of deflections along column line D

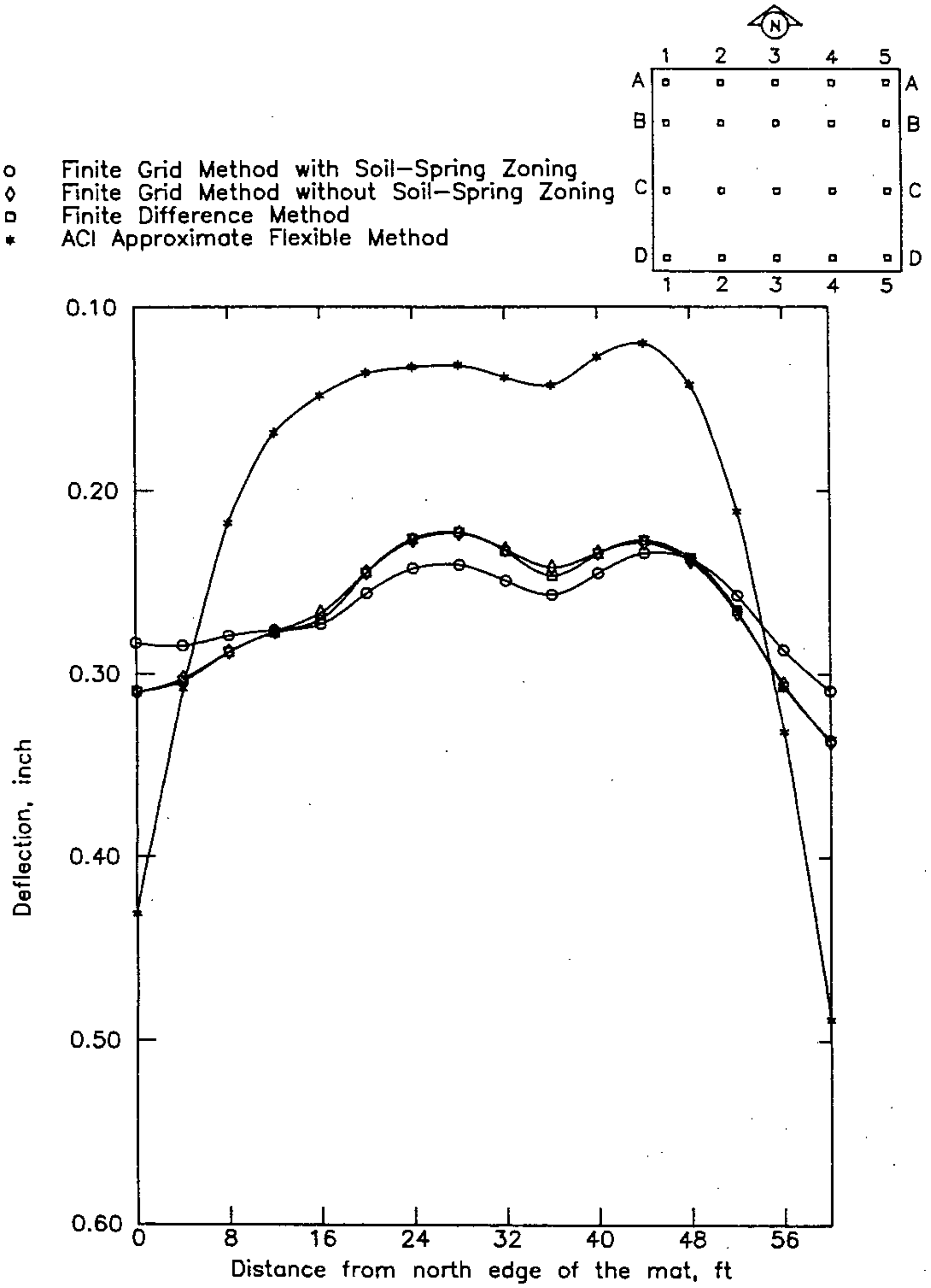


Fig.D.8 Comparison of deflections along column line 1

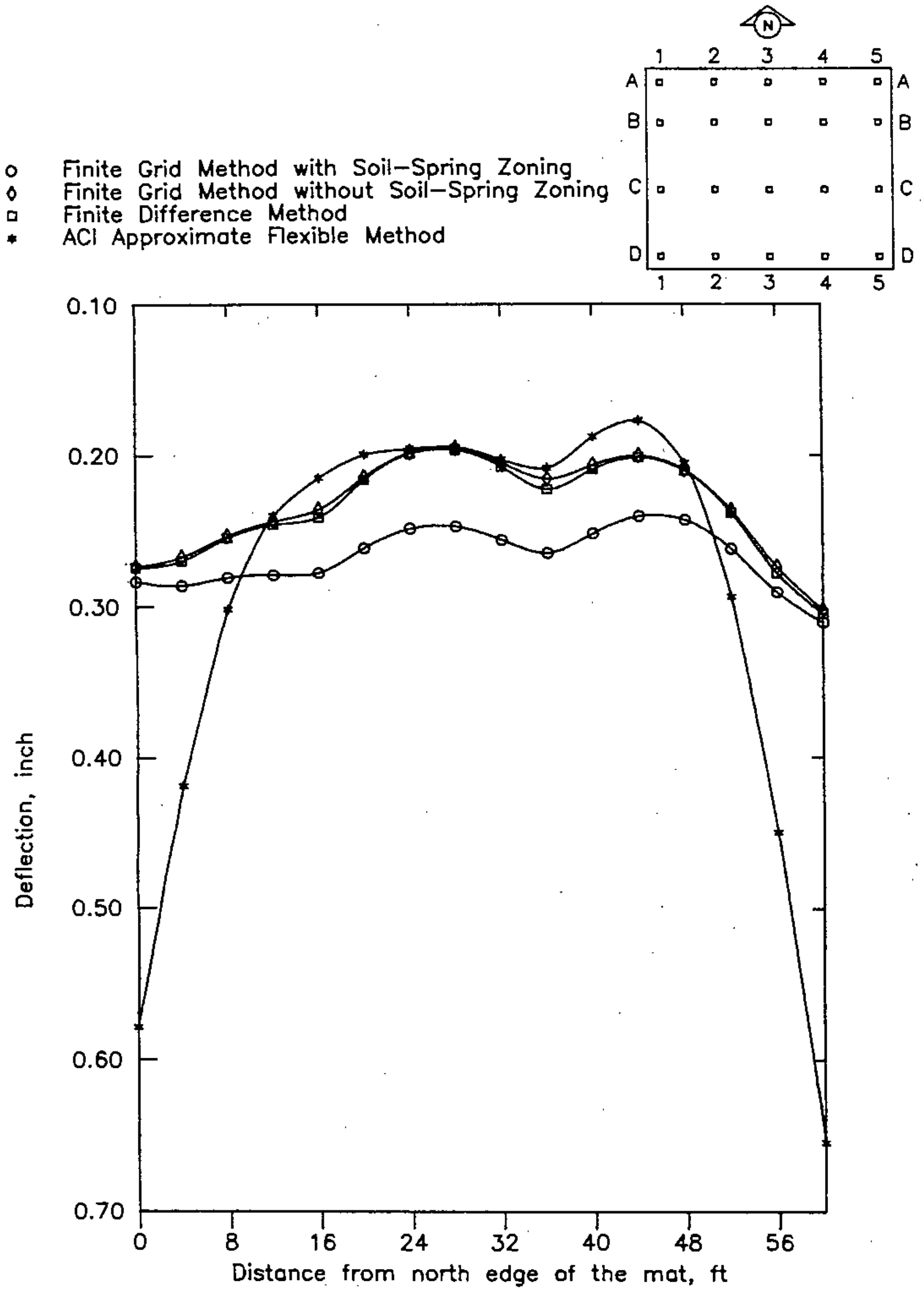


Fig.D.9 Comparison of deflections along column line 2

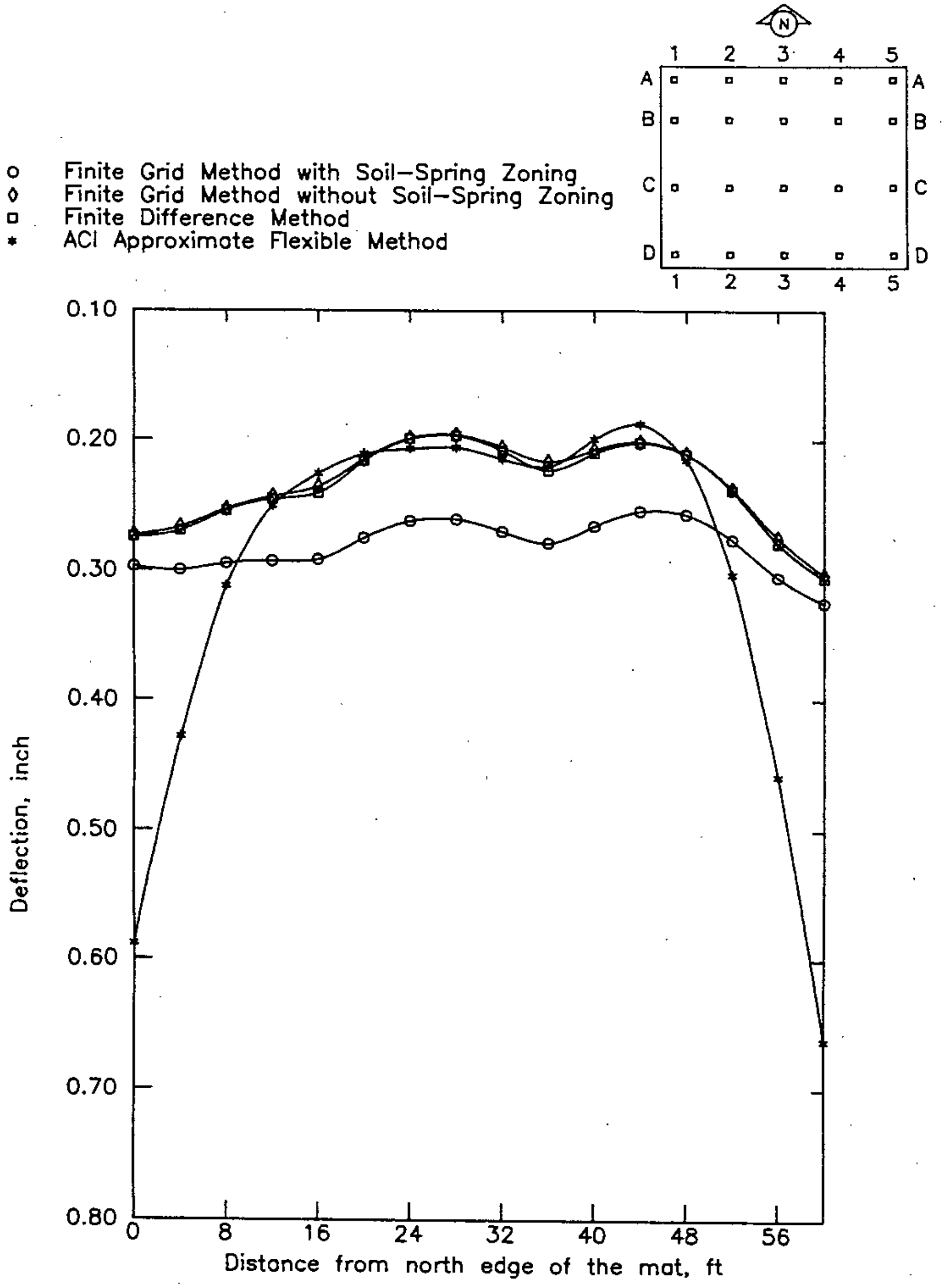


Fig.D.10 Comparison of deflections along column line 3

বাংলাদেশ প্রকৌশল বিশ্ববিদ্যালয়
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